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SUSY quivers, intersecting branes and the modest hierarchy problem

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Abstract: We present a class of chiral non-supersymmetric $D = 4$ field theories in which quadratic divergences appear only at two loops. They may be depicted as "SUSY quivers" in which the nodes represent a gauge group with extended e.g., $N = 4$ SUSY whereas links represent bifundamental matter fields which transform as chiral multiplets with respect to *different* $N = 1$ subgroups. One can obtain this type of field theories from simple D6-brane configurations on type IIA string theory compactified on a six-torus. We discuss the conditions under which this kind of structure is obtained from D6-brane intersections. We also discuss some aspects of the effective low-energy field theory. In particular we compute gauge couplings and Fayet-Iliopoulos terms from the Born-Infeld action and show how they match the field theory results. This class of theories may be of phenomenological interest in order to understand the *modest hierarchy problem* i.e., the stability of the hierarchy between the weak scale and a fundamental scale of order 10-100 TeV which appears e.g. in low string scale models. Specific D-brane models with the spectrum of the SUSY Standard Model and three generations are presented.

Keywords: D-branes, Supersymmetry Phenomenology, Beyond Standard Model, Compactification and String Models.

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1. Introduction

One of the most relevant properties of Dp-branes [1] in string theory is the fact that they localize gauge (and matter) interactions on a (p+1)-dimensional submanifold of the full higher dimensional space-time. This property opens the way to the generation of "mixed symmetry field theories" with different subsectors (coming from different localized sectors) with different symmetries and supersymmetries. Thus, depending from which sector of a given brane configuration a field theory is originated, one gets different symmetries and properties. One can envisage, for example, a field theory in which the pure gauge sector has $\mathcal{N}=4$ supersymmetry and some other massless fields respect only a subgroup or none of those supersymmetries. Whereas writing such type of lagrangians from scratch would look sort of contrived, from the string-theory and D-brane point of view they turn out to appear naturally.

Supersymmetry ($\mathcal{N} = 1$) has been applied to phenomenology in the last two decades due to the fact that quadratic divergences which may destabilize the hierarchy of scales are canceled. It is of obvious interest the search for $\mathcal{N} = 0$ theories in which there is some level of suppression of quadratic divergences. In particular, it has been recently realized that the scale of fundamental physics could be anywhere between, say 1-TeV and the Planck scale M_p [2]–[5]. The largeness of the Planck scale would then be an artifact of the presence of large extra dimensions or warp factors of the metric [6]. Thus it has been put forward the idea that the weak scale could be associated to the string scale [3, 4, 5]. On the other hand the SM works so well that it is difficult to make the string scale lighter than a few TeV without entering into conflict with experimental data (see e.g. ref. [7] for a nice physical view of the problem). Furthermore, the absence of any exotic source of flavour changing neutral currents (FCNC) would be most easily guaranteed if the string scale was postponed to scales of order 10-100 TeV. But if one has $M_s \sim 10 - 100\,\mathrm{TeV}$, we will have to explain the relative smallness of the weak scale $M_Z = 90\,\mathrm{GeV}$ compared to the string scale. This we call the "modest hierarchy problem".

An obvious cure to this problem is the full suppression of quadratic divergences offered by standard $\mathcal{N} = 1$ SUSY theories. On the other hand that is much more than what we actually need. To make a mass hierarchy of three orders of magnitude sufficiently stable it is enough to have absence of quadratic divergences up to one-loop, not at all loops. In the present paper, we present a class of chiral non-supersymmetric D=4 theories in which quadratic divergences only appear at two loops. We will call these models "quasi-supersymmetric" (Q-SUSY) because different subsectors of the theory respect different $\mathcal{N} = 1$ supersymmetries, but the complete lagrangian has $\mathcal{N} = 0$.[1] We think that this class of theories are interesting in their own right. Furthermore they offer a solution to the "modest hierarchy problem" described above. Absence of quadratic divergences up to one-loop would be enough to maintain a hierarchy between a string scale of order 10-100 TeV and the weak scale. This class of Q-SUSY gauge theories may be naturally obtained in a string context. One can show explicit string constructions with intersecting [8] D-branes wrapping a 6-torus [9]–[12] giving rise to such type field theories as their low-energy limit. We will be in fact able to construct concrete D6-brane models with a massless content quite analogous to that of the Minimal Supersymmetric Standard Model.

The class of models that we discuss in detail have a pure gauge sector respecting $\mathcal{N} = 4$ supersymmetry with a gauge group $\prod_i \mathrm{U}(M_i)$. In addition there are scalars and bosons multiplets transforming under bifundamental representations like $(M_i, \overline{M}_{i+1})$. However, each i^{th} such multiplet transforms as a chiral multiplet under a different $\mathcal{N} = 1$ subgroup of the $\mathcal{N} = 4$. Thus, as a whole the theory respects no supersymmetry at all. However, the spectrum remains Bose-Fermi degenerate up to one-loop and it is only broken at two loops by diagrams involving different chiral multiplets of different $\mathcal{N} = 1$ supersymmetries. One can obtain such type of field theories from simple D6-brane configurations on type IIA string theory compactified on a six torus $T^6 = T^2 \times T^2 \times T^2$. The gauge group of D6-branes

[1] It would be appropriate (particularly from the brane point of view) to call these theories *locally supersymmetric* but this may lead to confusion with gauged supersymmetry, i.e., supergravity.

on T^6 respects $\mathcal{N} = 4$ supersymmetry and has a U(M) symmetry. On the other hand if we wrap three of the D6-brane coordinates each one around one of the three T^2 tori, in general different stacks of D6-branes will intersect [9]–[12]. At the intersections chiral fermions in bifundamental representations appear. In addition massless scalars do also appear for certain intersecting angles. Thus the above structure of Q-SUSY field theories is obtained. In fact we found the structure of Q-SUSY models while studying the SUSY properties of the toroidal intersecting models of ref. [12]. Recently ref. [13] appeared in which a class of models with n-loop-suppression of quadratic divergences was presented. Although in our models there is a two-loop suppression of quadratic divergences, the models we find are essentially different in several respects.

The structure of the present paper is as follows. In the next section we describe the idea of quasi-supersymmetric field theories in general and present a graphic (SUSY quiver) way to describe some of their properties. In section 3 we briefly recall toroidal and orientifold compactifications of type II string theory with wrapping D6-branes intersecting at angles. These systems provide us with specific string constructions of Q-SUSY field theories. In section 4 we describe the conditions under which those theories give rise to Q-SUSY brane configurations. For certain complex structure values and brane wrapping numbers the property of Q-SUSY is obtained. RR tadpole cancellation conditions turn out to be very constraining and one can see that it is impossible to get full $\mathcal{N} = 1$ SUSY models, only Q-SUSY models may be obtained from D6-branes wrapping a six torus. However we will show that one can construct certain brane models in which a subsector of the theory has $\mathcal{N} = 1$ invariance with other $\mathcal{N} = 0$ subsectors acting as some sort of "hidden sectors" of the theory.

Some aspects of the effective field theory of the Q-SUSY brane configurations are discussed in section 5. In particular we compute the gauge coupling constants and Fayet-Iliopoulos terms from the DBI action and show how they match the field theory expectations from holomorphicity. These results apply as well to $\mathcal{N} = 1$ supersymmetric configurations like those studied in [14]. Small variations of the complex structure around the $\mathcal{N} = 1$ (or Q-SUSY) points give rise to such Fayet-Iliopoulos terms. Whereas in the $\mathcal{N} = 1$ case the turning on of a FI-term induces gauge symmetry breaking but not SUSY-breaking, in the case of Q-SUSY brane configurations the FI-terms may induce in general both gauge and local SUSY breaking.

Although, as expected, the NS complex structure potential in the case of Q-SUSY brane configurations does not vanish (leading to NS-tadpoles), we show that it has a simplified form compared to generic toroidal non-Q-SUSY configurations. In particular we show that such scalar potential is linear in the complex structure fields and that in particular cases the $D = 4$ dilaton tadpole vanishes as a consequence of RR-tadpole cancellations.

We present some specific models in section 6. In particular we present a model with the spectrum of the SUSY SM, three quark-lepton generations and a doubled Higgs sector. In this realistic example quadratic divergences only appear at two-loops, providing an specific example of an stabilized M_s/M_Z hierarchy as mentioned above. In the present context the SM Higgs mechanism has a nice geometrical interpretation as a recombination of the three branes supporting the gauge group $\mathrm{U}(2)_L \times \mathrm{U}(1)$ into a single brane. We present some general comments and conclusions in section 7.

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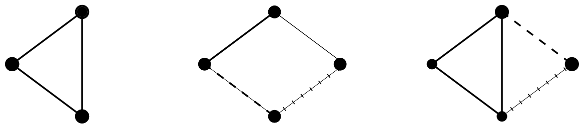

a) b) c)

Figure 1: Some quiver-like graphs representing SUSY and Q-SUSY field theories: a) A SUSY theory with three gauge factors; b) A Q-SUSY model with four group factors and chiral bifundamental fermions filling SUSY multiplets with respect to four different SUSY's; c) A Q-SUSY model with four group factors and fermions inside SUSY multiplets with respect to three different SUSY's.

2. Quasi-supersymmetric models

A Q-SUSY field theory is one in which different subsectors of the theory respect some $\mathcal{N}=1$ supersymmetry but different sectors clash with each other so that the complete lagrangian has no surviving supersymmetry. The gauge interactions have extended $\mathcal{N}=4$ (or $\mathcal{N}=2$) supersymmetry whereas the chiral matter fields respect only some of the $\mathcal{N}=1$ subgroups. Thus, for example, a possible general structure for the lagrangian is as follows:[2]

$$L = L(\mathcal{N}=4) + \sum_i L_i(\mathcal{N}=1) \qquad i=1,2,3,4 \tag{2.1}$$

where i labels four independent supersymmetries inside $\mathcal{N}=4$. Here $L(\mathcal{N}=4)$ is the $\mathcal{N}=4$ lagrangian of a number of group factors, i.e. $\prod_a \mathrm{U}(N_a)$. On the other hand each of the $L_i(\mathcal{N}=1)$ terms respect a different $\mathcal{N}=1$ subgroup of the full $\mathcal{N}=4$. There will be chiral multiplets $\Phi_a(\phi_a,\psi_a)$ with respect to the four different $\mathcal{N}=1$'s. They will transform as bifundamentals under the $\prod_a \mathrm{U}(N_a)$ gauge group

$$\Phi_a = (N_a, \overline{N}_{a+1}) \tag{2.2}$$

or

$$\Phi_a = (N_a, N_{a+1}) \tag{2.3}$$

Each of these chiral multiplets may come in several copies. Such theories will in general be chiral and one will have to insure the cancellation of anomalies. In the case of models obtained from D-brane constructions that will be guaranteed by cancellation of RR-tadpoles. Note that the a^{th} chiral multiplet couples to some i^{th} gaugino but not to the other three $\mathcal{N}=4$ gauginos. So each of the boson-fermion multiplets form a chiral multiplet with respect to a different $\mathcal{N}=1$ subgroup. It is clear that from the point of view of one of the $\mathcal{N}=1$'s the other break explicitly supersymmetry since its gauginos do not connect the other scalars into their fermionic partners. Thus the complete theory will have no supersymmetry unbroken, all supersymmetries are broken explicitly.

One can represent graphically this class of theories in a quiver-like notation, as in the examples depicted in figure 1. Here the blobs denote $\mathrm{U}(N_a)$ factors with $\mathcal{N}=4$ supersymmetry. The links represent the bifundamental chiral multiplets Φ_a. The different

[2] We concentrate in this class because it is the one which appears most easily in explicit D-brane toroidal models.

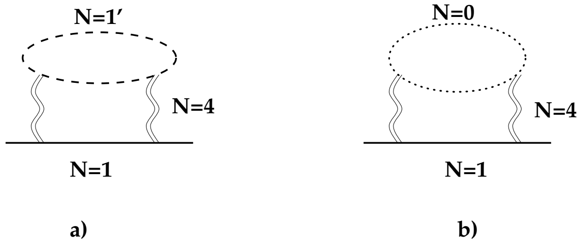

Figure 2: First non-vanishing loop contributions to the masses of scalar fields in a Q-SUSY model: a) Quadratic divergent contribution present when the upper loop contains *massless* fields respecting different supersymmetries than those of the fields below; b) Contribution coming from possible *massive* non-supersymmetric states in the upper loop.

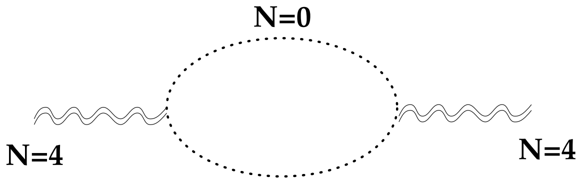

Figure 3: One-loop contribution to the masses of gauginos. Heavy non-supersymmetric states circulate in the loop.

style of the lines indicate that the boson and fermion in the multiplet are partners with respect to a different $\mathcal{N}=1$ SUSY inside $\mathcal{N}=4$. Analogous quiver-like structure of gauge theories has arisen in recent years both in the context of string theory (see e.g. [15]) and field theory[16].

It is clear from the structure of this class of theories that, since the gauge multiplet conserves all four SUSY's, up to one loop the usual no-renormalization theorems apply for each of the four $\mathcal{N}=1$'s independently. Thus quadratic divergences will be absent up to one-loop. The first loop corrections (figure 2(a)) to the masses of the scalars ϕ_a will appear at two-loop order

$$m_a^2 \propto \left[\left(\frac{\alpha_a}{4\pi}\right)^2 + \left(\frac{\alpha_{a+1}}{4\pi}\right)^2\right] M^2 \tag{2.4}$$

where M will be some cut-off scale (the string scale in the D-brane examples discussed below). As we will see later on, specific D-brane realizations of the Q-SUSY structure have in addition sectors which are *massive* and respect no supersymmetry at all. These $\mathcal{N}=0$ sectors in general decouple but do contribute in loops to the effective action of the massless fields. In particular those massive fields also contribute in two-loop order (figure 2(b)) to the masses of the scalars and at one-loop give masses to the fermion fields in the $\mathcal{N}=4$ gauge multiplets (figure 3)). Adjoint scalars get their masses from diagrams analogous to those in figure 2(b). Note that in the absence of these massive $\mathcal{N}=0$ sectors only the scalars would get (two-loop) masses but not the gauginos.

We thus see that the scalars in this class of theories have a two-loop protection against quadratic divergences. This in general will not be sufficient to solve the large hierarchy problem between the weak-scale and the Planck scale.[3] However it has been recently realized that the scale of new fundamental physics may well be much below, at scales of order 10-100 TeV. If this is the case we will have to explain why the weak scale is 2 or 3 orders of magnitude below this new scale. In this connection the partial protection against quadratic divergences of Q-SUSY models could be of some interest. We will present later on some semi-realistic D-brane models making use of this possibility and leave further phenomenological applications of this idea to a separate publication.

The idea of Q-SUSY field theories may be considered independently of any string theory argument. However it is in the realm of string theory and brane physics that it looks more natural. In the rest of this paper we are going to present explicit realizations of the idea in terms of type II D6-branes wrapping a six-torus. In section 6 some explicit D-brane models are presented.

3. D6-branes wrapping intersecting cycles in toroidal and orientifold compactifications

Let us describe a simple string setting where the idea of Q-SUSY models can easily be realized. Consider type IIA string theory compactified on a six dimensional manifold $\mathcal{M}$. When constructing a brane configuration, our building blocks will consist of D6-branes filling four-dimensional Minkowski space-time and wrapping homology 3-cycles of $\mathcal{M}$. A specific configuration of branes will be given by K stacks of branes, each stack containing N_a coincident D6-branes wrapping the 3-cycle $[\Pi_a] \in H_3(\mathcal{M}, \mathbb{Z})$, $(a = 1, \ldots, K)$. The gauge group of such configuration will be given by $\prod_a \mathrm{U}(N_a)$. There is a chiral fermion [8] living at each four-dimensional intersection of two branes a and b, transforming in the bifundamental representation of $\mathrm{U}(N_a) \times \mathrm{U}(N_b)$. The intersection number of these two branes, $I_{ab} \equiv \Pi_a \cdot \Pi_b$ is a topologically invariant integer whose modulus gives us the multiplicity of such massless fermionic content and its sign depends on the chirality of such fermions.

Any consistent configuration has to satisfy some conditions related to the propagation of RR massless closed string fields on the compact manifold $\mathcal{M}$. These are the RR tadpole cancellation conditions which can be expressed in a very simple way in the context of type IIA D6-branes wrapping 3-cycles. Namely, the sum of the homology cycles where the branes wrap must add up to zero [10, 17]

$$\sum_a N_a [\Pi_a] = 0\,. \tag{3.1}$$

When considering theories where additional sources for RR charges appear, such as orientifold compactifications, this topological sum of RR charges must cancel the charge induced by the O6-plane. It can be easily seen that RR tadpoles conditions directly imply

[3] See however footnote 17.

the cancellation of non-abelian $\mathrm{SU}(N_a)^3$ anomalies. They also imply, by the mediation of a Green-Schwarz mechanism, the cancellation of mixed non-abelian and gravitational anomalies [10, 12, 14].

A particularly simple subfamily of the configurations described above consist of taking $\mathcal{M}$ as a factorizable six-torus: $\mathcal{M} = T^2 \times T^2 \times T^2$. We can then further simplify our configurations by considering branes wrapping factorizable 3-cycles, that is, cycles that can be expressed as products of 1-cycles on each T^2. This setup and some orbifold and orientifold variations have previously been considered in [9]–[12], [18, 14, 19], where explicit configurations have been constructed.[4] In this case the homology 3-cycle Π_a can be expressed as

$$[\Pi_a] \equiv \left[\left(n_a^1, m_a^1\right), \left(n_a^2, m_a^2\right), \left(n_a^3, m_a^3\right) \right], \tag{3.2}$$

where n_a^i, m_a^i are integers describing wrapping numbers around the tori cycles. The intersection number takes the simple form

$$I_{ab} = \prod_{i=1}^{3} \left(n_a^i m_b^i - m_a^i n_b^i \right), \tag{3.3}$$

whose modulus gives us the number of chiral fermions at the intersection. An interesting variation of toroidal compactifications consist of modding out our six-torus by the $\mathbb{Z}_2$ orientifold group action $\{1, \Omega\mathcal{R}\}$, where Ω is the world-sheet parity and $\mathcal{R}$ is a reflection on the vertical coordinates x^5, x^7, x^9. This system naturally arises by considering type I string theory compactified on T^6, where D9-branes with fluxes appear, and performing a T-duality on these vertical coordinates [8, 9, 11, 25].[5] These orientifold compactifications have several new features compared to the toroidal ones. First, an O6-plane appears in the compactification, wrapping a 3-cycle $[\Pi_{ori}]$. Second, to each stack of branes a we must add its image under the orientifold group generator $\Omega\mathcal{R}$. Thus, to each sector $D6_a$ we must add a mirror sector $\Omega\mathcal{R}D6_a$ or $D6_{a^*}$. When dealing with square tori, tadpoles read

$$\sum_a N_a \left([\Pi_a] + [\Pi_{a^*}] \right) = 32[\Pi_{\mathrm{ori}}] \tag{3.4}$$

and it can easily be seen by performing the above mentioned T-duality that we are left with an O6-plane on the 3-cycle $[(1,0),(1,0),(1,0)]$. If a brane is wrapping the homology cycle (3.2), then its mirror brane is given by

$$[\Pi_{a^*}] \equiv [(n_a^1, -m_a^1), (n_a^2, -m_a^2), (n_a^3, -m_a^3)]. \tag{3.5}$$

In a configuration composed by factorizable branes, then, the tadpole conditions read [9]

$$\sum_a N_a n_a^1 n_a^2 n_a^3 = 16,$$
$$\sum_a N_a m_a^1 m_a^2 n_a^3 = 0,$$

[4]For some related constructions involving branes at angles other than D6-branes see [10, 20, 21]. For some work concerning phenomenological aspects of these constructions see [22, 12, 18, 23, 24].

[5]See also [26, 27, 28]

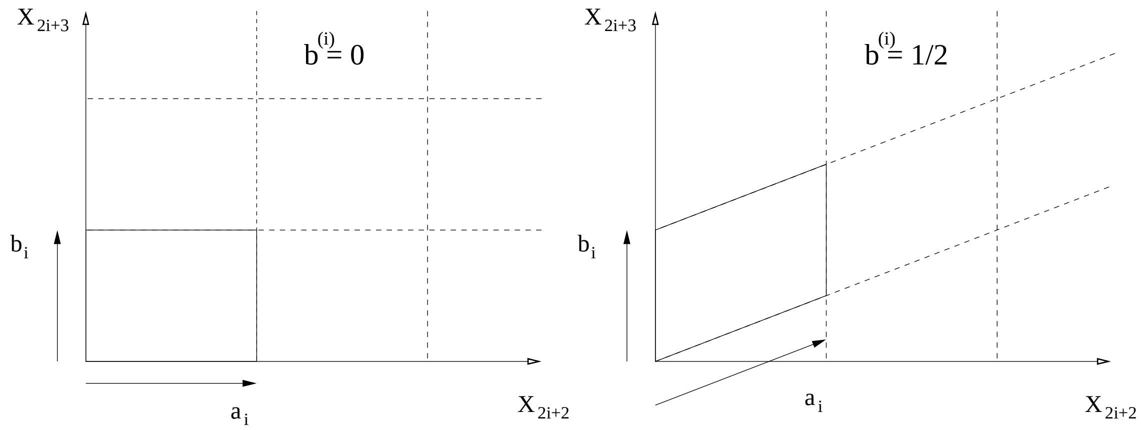

Figure 4: Allowed toroidal lattices in orientifold models.

$$
\begin{aligned}
\sum_a N_a m_a^1 n_a^2 m_a^3 &= 0\,, \\
\sum_a N_a n_a^1 m_a^2 m_a^3 &= 0.
\end{aligned}
\tag{3.6}
$$

Notice that only four conditions appear in (3.6), in contrast with the eight conditions appearing in the toroidal case (see [10]). This signals that only four four-dimensional RR fields are relevant for tadpoles under this orientifold compactification. Similarly, some of the NSNS fields that describe the geometry of the T^6 are no longer dynamical. This is because not any complex structure is well defined under the identification $\Omega\mathcal{R}$, which only allows square tori and a special kind of tilted tori (see figure 4). In the T-dual picture, this translates into a B-field non-invariant under Ω, which only allows discrete values $b = 0, 1/2$ [28, 11]. In order to easily describe these configurations with non-vanishing b-flux, we define effective wrapping numbers, which can now take semi-integer values

$$
(n_a^i, m_a^i)_{\text{eff}} \equiv (n_a^i, m_a^i) + b^{(i)}(0, n_a^i)\,, \tag{3.7}
$$

where $b^{(i)}$ stands for the value of b in the i^{th} T^2. Expressing our factorizable D6-branes in terms of these fractional wrapping numbers makes (3.5) and (3.6) still valid in the presence of non-trivial b-flux. Notice, however, that the O6-plane will now lie in the 3-cycle $[(1/\beta^1, 0)_{\text{eff}}, (1/\beta^2, 0)_{\text{eff}}, (1/\beta^3, 0)_{\text{eff}}]$, where $\beta^i = 1 - b^{(i)}$.[6]

Let us finally point out that considering orientifold models introduces new sectors arising from open strings stretching between branes. To the previous $D6_aD6_a$ sector, giving rise to unitary gauge group $\mathrm{U}(N_a)$,[7] and the $D6_aD6_b$ sector, where massless fermions in the bifundamental $(N_a, \overline{N}_b)$ live, we must add the $D6_aD6_{b^*}$ sector, with massless fermions in the (N_a, N_b) representation, and the $D6_aD6_{a^*}$, where 'exotic matter' transforming in the symmetric and antisymmetric representation of $\mathrm{U}(N_a)$ may appear (for more details see [9, 12]).

[6]From now on we will suppress the subindex 'eff', considering always fractional wrapping numbers.

[7]In orientifold models, also $\mathrm{SO}(N)$ and $\mathrm{USp}(N)$ gauge groups may appear if a brane a is its own image under the orientifold action, but this will not usually be the case in our constructions.

4. Systems of branes preserving a supersymmetry

Let us consider IIA D6-branes wrapping homology 3-cycles on a six dimensional manifold $\mathcal{M}$. If we consider an isolated brane in a given homology class $[\Pi_a] \in H_3(\mathcal{M}, \mathbb{Z})$, this brane will tend to minimize its tension by wrapping the submanifold of minimum length inside this homology class.[8] A particular class of volume-minimizing submanifolds are Special lagrangian submanifolds (SLAGs) which appear naturally as calibrated objects in the geometry of Calabi-Yau compactifications. These manifolds have been discussed recently in the literature [29]–[32]. The precise form, and even the existence of these SLAGs, depends on the specific point of the moduli space of complex structures of $\mathcal{M}$ we are sitting on. A brane wrapping a Special lagrangian submanifold will preserve a supersymmetry, thus studying SLAGs yields an interesting family of BPS states in a particular compactification. A full configuration of D6-branes may, however, be composed of branes that preserve different supersymmetries, thus leading to a non-supersymmetric system.

4.1 System of two branes

In general, we can detect whether two branes $D6_a$ and $D6_b$ preserve a common supersymmetry by looking at the spectrum living at the $D6_aD6_b$ sector. A particularly simple setting consist of D6-branes wrapping factorizable cycles of a $T^2 \times T^2 \times T^2$. By factorizable we mean branes wrapping a 1-cycle on each T^2, so we are actually considering a sublattice of the whole $H_3(T^6, \mathbb{Z})$. The SLAGs corresponding to these factorizable cycles are the product of three straight lines wrapping the corresponding 1-cycle on each torus, thus yielding a flat 3-submanifold of T^6. Each of these branes preserves the maximum number of supersymmetries, which is $\mathcal{N} = 4$.

We may consider a system of two branes: $D6_a$ and $D6_b$, both wrapping a factorizable cycle on our six-dimensional torus. As both branes make an angle ϑ^i_{ab} on the i^{th} torus, we can describe the spectrum in the $D6_aD6_b$ sector by introducing a four-dimensional twist vector v_ϑ [10]

$$v_\vartheta = \left(\vartheta^1_{ab}, \vartheta^2_{ab}, \vartheta^3_{ab}, 0\right), \tag{4.1}$$

where the fourth component describes the non-compact complex dimension in light-cone gauge. Here we are considering ϑ^i_{ab} in units of π, so that $-1 \leq \vartheta^i_{ab} \leq 1$. The states living in this sector can be described by the sum of vectors $r + v_\vartheta$, where $r \in (\mathbb{Z} + \nu)^4$. The GSO projected states are those that satisfy $\sum_i r^i = \text{odd}$, both for the R sector ($\nu = \frac{1}{2}$) and the NS sector ($\nu = 0$). The mass of these states is given by [33, 10]

$$\alpha' M^2_{ab} = \frac{Y^2}{4\pi\alpha'} + N_{bos}(\vartheta) + \frac{(r + v_\vartheta)^2}{2} - \frac{1}{2} + E_{ab}, \tag{4.2}$$

where $N_{bos}(\vartheta)$ is the contribution from the bosonic oscillators and E_{ab} is the vacuum energy:

$$E_{ab} = \sum_{i=1}^{3} \frac{1}{2}|\vartheta^i|(1 - |\vartheta^i|) \tag{4.3}$$

[8] This is strictly true only when no B or F fluxes are turned on, so the minimization of the DBI action is equivalent to the minimization of the volume.

Given a generic twist vector v_ϑ with non-trivial angles ϑ^i_{ab}, we will always find a massless fermion in the R sector, described by the vector

$$r_R = \frac{1}{2}\left(-s\left(\vartheta^1_{ab}\right), -s\left(\vartheta^2_{ab}\right), -s\left(\vartheta^3_{ab}\right), \prod_{i=1}^{3} s\left(\vartheta^i_{ab}\right)\right), \tag{4.4}$$

where $s(\vartheta^i_{ab}) \equiv \mathrm{sign}(\vartheta^i_{ab})$. This massless fermion will live at the submanifold where both branes intersect, which is generically one or several points in the compact space (times the full four-dimensional Minkowski space). The fourth component of r, which describes the four-dimensional Lorentz quantum numbers of the state, is fixed by GSO projection, and it is easy to see that its sign agrees with that of the intersection number I_{ab}.

Looking at the lightest states coming from the NS sector we find four different scalars with masses given by [10]:

State ($r_{\rm NS}$)	**Mass^2**
$(-s(\vartheta^1), 0, 0, 0)$	$\alpha' M^2 = \frac{1}{2}(-\vert\vartheta^1\vert + \vert\vartheta^2\vert + \vert\vartheta^3\vert)$
$(0, -s(\vartheta^2), 0, 0)$	$\alpha' M^2 = \frac{1}{2}(\vert\vartheta^1\vert - \vert\vartheta^2\vert + \vert\vartheta^3\vert)$
$(0, 0, -s(\vartheta^3), 0)$	$\alpha' M^2 = \frac{1}{2}(\vert\vartheta^1\vert + \vert\vartheta^2\vert - \vert\vartheta^3\vert)$
$(-s(\vartheta^1), -s(\vartheta^2), -s(\vartheta^3), 0)$	$\alpha' M^2 = 1 - \frac{1}{2}(\vert\vartheta^1\vert + \vert\vartheta^2\vert + \vert\vartheta^3\vert)$ (4.5)

Notice that these four scalars may be massive, massless or even tachyonic, depending on the relative angles both branes make. In general, when having one of these four scalars massless, the sector $D6_aD6_b$ will present a degeneracy in mass between bosonic and fermionic states, all the spectrum of particles arranging themselves into D=4, $\mathcal{N}=1$ supermultiplets. This signals that, for this particular choice of complex structure, our combined system of D6-branes preserves a common SUSY. It may also happen that two (or even four) of the scalars become massless, yielding a D=4 $\mathcal{N}=2$ ($\mathcal{N}=4$) spectrum [12, 25].

The supersymmetry preserved for such system can be characterized by a vector $\tilde{r} \in (\mathbb{Z}+1/2)^4$ with opposite GSO projection, defined by $\tilde{r} \equiv r_{\rm NS} - r_R$, where $r_{\rm NS}$ corresponds to a massless scalar. Indeed, we can associate to $\tilde{r}$ a SUSY generator $Q_{\tilde{r}}$, that takes us from a massless fermionic state to a massless bosonic state:

$$Q_{\tilde{r}} | r + v_\vartheta >_R = | r + \tilde{r} + v_\vartheta >_{\rm NS} \ . \tag{4.6}$$

This connection between bosonic and fermionic states will not only hold at the massless level, but will also be true for any massive supermultiplet in the $D6_aD6_b$ sector.

4.2 System of several branes

When facing the problem of building a chiral theory arising from the low energy description of a fully-fledged compactification of branes intersecting at angles, usually more than two stacks of branes have to be considered. This is mainly due to the fact that, in order to have a consistent anomaly-free 4D theory, some constraints have to be satisfied, namely the RR tadpole cancellation conditions. As we have seen in section 3, RR tadpoles cancellation translates into a topological restriction on the sum of the homology classes where the D6-branes wrap. Briefly stated, this sum has to be equal to the homology class of the O6-plane, if present.

When studying supersymmetric compactifications, however, we realize that the condition for a pair of factorizable branes to preserve a SUSY depends on the angles they make on each of the tori. Supersymmetry, then, turns out to be a geometrical question, rather than a topological one. By varying the complex structure of the manifold where the D6-branes wrap, we can go from a SUSY configuration to a non-SUSY one. A natural question is whether we can go the other way round, that is, if any generic compactification allows for a nontrivial chiral SUSY model by suitably changing the complex structures.

This question can be made more precise. It is well known that two factorizable D6-branes at angles preserve a supersymmetry if they are related by a rotation which belongs to SU(3) [8, 34]. Now, this SU(3) rotation can be embedded in several ways into the tangent space rotation group $SO(6) \simeq SU(4)$. Let us describe the relative position between two factorizable branes a and b by a rotation matrix acting on the compact complex coordinates $z^i = x^{2i+2} + ix^{2i+3}$ that parametrize each of the tori T_i^2 $(i = 1, 2, 3)$, as done in [34].

$$R_{ab} : \begin{pmatrix} z^1 \\ z^2 \\ z^3 \end{pmatrix} \mapsto \begin{pmatrix} e^{i\pi\vartheta^1} & 0 & 0 \\ 0 & e^{i\pi\vartheta^2} & 0 \\ 0 & 0 & e^{i\pi\vartheta^3} \end{pmatrix} \cdot \begin{pmatrix} z^1 \\ z^2 \\ z^3 \end{pmatrix} \tag{4.7}$$

In general, R belongs to a U(3) subgroup of SO(6) that preserves the complex structure of T^6. Since we have chosen it to relate two factorizable branes which are 1-cycles on each T^2, it is also diagonal. The conditions for preserving a SUSY can be read from the previous discussion, in terms of the masses (4.5). This translates into a restriction on R, which is $\frac{1}{2}\sum_i \vartheta^i \epsilon_i \in \mathbb{Z}$ for some phases $\epsilon_i = \pm 1$.[9] Notice that this implies $R \in SU(3)$ (or some isomorphic embedding of this group into SO(6)).

This fact can easily be applied to a full configuration of branes, where the same SUSY will be preserved by any of the branes if they are related to each other by rotations of the same SU(3). So, in order to see if we have a supersymmetric configuration of factorizable branes, we only have to compute the rotations R_{ab} relating each pair of D6-branes a and b and check that they all belong to the same SU(3) subgroup. This will mean, in turn, that the vector $\tilde{r}$ at each intersection is the same.

4.3 SUSY and Q-SUSY orientifold systems

There is a simple way to show that chiral supersymmetric configurations are impossible to obtain in toroidal models of D6-branes considered in [10]. Indeed, it is very easy to construct RR tadpole-free configurations of this kind of compactifications. However, having

[9]More strictly speaking, this restriction comes from imposing $R^2_{ab} Q_{\tilde{r}} = Q_{\tilde{r}}$ for some $\tilde{r}$, where $Q_{\tilde{r}}$ is one of the ten-dimensional spinor states appearing in type IIA string theory. See [35, 9].

supersymmetric configurations would automatically imply the cancellation of NSNS tadpoles, which are proportional to the sum of tensions of the branes. As mentioned in [18], this can easily be seen by the dimensional reduction of the whole Dirac-Born-Infeld action for every D6-brane.

$$\begin{aligned} \mathcal{S}_{DBI} &= -T_6 \sum_a \int_{D6_a} d^{p+1}\xi e^{-\phi} \sqrt{\det(G+\mathcal{F}_a)} \\ &= -\int_{\mathcal{M}_4} d^4x \, \frac{T_6}{\lambda} \sum_a \|l_a\| \, . \end{aligned} \tag{4.8}$$

Here, T_6 is the tension of the D6-branes from the general formula $T_p = (2\pi)^{-p}\alpha'^{\,-\frac{p+1}{2}}$ [34]. λ is the string coupling and $\|l_a\|$ stands for the volume of the 3-manifold where the brane lies inside the compact space T^6. Since this quantity only vanishes if the sum of tensions is null, that is if no branes are present, we deduce that the only way to have a supersymmetric configuration is just having type IIA string theory compactified on T^6, with no branes at all. In order to avoid this result, we can perform some orientifolding on our configuration. This will introduce some negative contribution to the NS potential, coming from the negative tension of the O6-plane

$$V = \frac{T_6}{\lambda}\left(\sum_a \|l_a\| - \|l_{\rm ori}\|\right) . \tag{4.9}$$

This allows us to have some brane content when canceling both RR and NSNS tadpoles. Notice, however, that this may not be good enough. If the O6-plane lies in just one factorizable 3-cycle of T^6, as in orientifold models considered in [9, 11, 12], then the supersymmetric configuration will only be attained when every D6-brane lies on top of the orientifold (or parallel to it), yielding just a T-dual version of type I string theory compactified on a torus. In order to attain a chiral supersymmetric configuration we should have, then, several O6-planes wrapping different cycles.[10] In [14] a family of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold models was presented, where the O6-plane wrapping a definite factorizable 3-cycle had to be completed by its images under the orbifold $\mathbb{Z}_2 \times \mathbb{Z}_2$ group action.

Adding some negative tension to (4.9) is not the only effect of considering an orientifold compactification. When constructing a toroidal model, a factorizable brane a by itself, being a $\frac{1}{2}$BPS state will preserve $\mathcal{N} = 4$ supersymmetry. In an orbifold or orientifold model, however, this brane has to share some supersymmetry with its images under the orientifold group action. Stated differently, the brane a has to share some supersymmetry with the O6-plane, that is, it should be related by a SU(3) rotation (4.7) with the O6-plane. If the O6-plane is wrapping a factorizable cycle, then it preserves, so to speak, $\mathcal{N} = 4$, and it can share a supersymmetry with a brane in many different ways. In the models presented in [14], however, the $\mathbb{Z}_2 \times \mathbb{Z}_2$ twist was already embedded as a discrete group of a definite SU(3) rotation group. This has two consequences. First, the ambient space will only preserve $\mathcal{N} = 1$ (as in usual Calabi-Yau orientifold theories). Second, if a brane is to preserve a SUSY with its image under the orientifold group, then it should be related to the

[10] Or, equivalently, a O6-plane wrapping a non-factorizable cycle.

O6-plane(s) by a rotation inside this same SU(3). In the explicit constructions presented in [14] this condition translated into $\vartheta^1 + \vartheta^2 + \vartheta^3 \in \mathbb{Z}$. Indeed, the branes considered in model building had the following twist vectors:

$$\begin{array}{ll} a \text{ branes}: & v_a = (0, \vartheta_a, -\vartheta_a) \\ b \text{ branes}: & v_b = (\vartheta_b, 0, -\vartheta_b) \\ c \text{ branes}: & v_c = (\vartheta_c, -\vartheta_c, 0) \end{array} \tag{4.10}$$

The supersymmetry preserved by any intersection of these branes with the O6-plane or any of its images under $\mathbb{Z}_2 \times \mathbb{Z}_2$ is given by the vector $\tilde{r} = \pm\frac{1}{2}(+,+,+,+)$. By transitivity, any intersection of branes also preserves this same supersymmetry. Notice that $\tilde{r}$ is the only spinor invariant under the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold group, so the only candidate SUSY to be preserved by a pair of branes. This argument seems general for any orientifold compactification with the O6-plane wrapping a non-factorizable cycle.

The setup presented in [14] allowed for the construction of an interesting class of chiral supersymmetric models of branes at angles, as was explicitly shown. However, the introduction of the orbifold twists is not harmless from the model building point of view and the massless chiral spectrum tends to yield particles with exotic quantum numbers. In this paper we will concentrate in the simpler case of toroidal orientifolds without orbifold twists, although some of our results, particularly those related to the effective lagrangian in section 5 will remain valid in the orbifold cases.

When considering an orientifold compactification where the O6-plane lies in a factorizable cycle we can consider a different possibility when constructing a "supersymmetric" model, which might be of phenomenological interest. Instead of asking to our branes to share the same SUSY $\tilde{r}$ with the O6-plane, we can relax this condition and ask them to preserve at least one SUSY with it, but not necessarily the same one for each brane. In this setup, a pair of branes intersecting at a point may, as well, share a SUSY, thus having a boson-fermion degenerate mass spectrum. If this happens for every pair of intersecting branes, then, for each massless fermion living at the intersections we will have a massless boson superpartner. Since each intersection preserves a different SUSY, we will effectively have $\mathcal{N} = 0$ when considering the full field theory at low energy. However, locally (at each intersection) particles will arrange as SUSY multiplets. This is an explicit D-brane realization of the Q-SUSY idea introduced in section 2.

Let us give an example realizing this idea. We will consider type IIA D6-branes wrapping factorizable cycles of a torus orientifolded by $\Omega\mathcal{R}$. In this particular setting, branes are allowed to preserve $\mathcal{N} = 2$ SUSY with the orientifold and their mirror images. We will consider all possible types of such "$\mathcal{N} = 2$ branes" in order to describe the different Q-SUSY structures they may lead to. A quite general configuration and its possibilities is illustrated by the brane content shown in table 1.

The six stacks of branes will give rise to a $\mathrm{U}(N_{a_1}) \times \mathrm{U}(N_{a_2}) \times \mathrm{U}(N_{b_1}) \times \mathrm{U}(N_{b_2}) \times \mathrm{U}(N_{c_1}) \times \mathrm{U}(N_{c_2})$ gauge group. In order to obtain a non-anomalous 4D theory tadpole cancellations should be fulfilled, imposing $N_{i_1} = N_{i_2} = N_i$, $i = a, b, c$. In order to cancel the orientifold charge (nnn tadpole) an additional stack of N_h branes parallel to the orientifold may be

N_i	(n_i^1, m_i^1)	(n_i^2, m_i^2)	(n_i^3, m_i^3)	v_i
N_{a_1}	$(1,0)$	$(1, m_a^2)$	$(1, m_a^3)$	$(0, \vartheta_a^2, \vartheta_a^3)$
N_{a_2}	$(1,0)$	$(1, m_a^2)$	$(1, -m_a^3)$	$(0, \vartheta_a^2, -\vartheta_a^3)$
N_{b_1}	$(1, m_b^1)$	$(1,0)$	$(1, m_b^3)$	$(\vartheta_b^1, 0, \vartheta_b^3)$
N_{b_2}	$(1, -m_b^1)$	$(1,0)$	$(1, m_b^3)$	$(-\vartheta_b^1, 0, \vartheta_b^3)$
N_{c_1}	$(1, m_c^1)$	$(1, m_c^2)$	$(1,0)$	$(\vartheta_c^1, \vartheta_c^2, 0)$
N_{c_2}	$(1, m_c^1)$	$(1, -m_c^2)$	$(1,0)$	$(\vartheta_c^1, -\vartheta_c^2, 0)$

Table 1: Example of D6-brane wrapping numbers giving rise to a "$\mathcal{N}=2$" Q-SUSY model. We are supposing $m_i^j \geq 0$, $i = a, b, c$, $j = 1, 2, 3$.

added. Having wrapping numbers $(1,0)(1,0)(1,0)$, it does not intersect with any other brane, so it can be considered as a hidden sector of the theory. The last tadpole condition reads then

$$2N_a + 2N_b + 2N_c + N_h = 16\,. \tag{4.11}$$

In the last column of table 1 we introduced the twist vector of each stack of branes, where $\vartheta_i^j = tg^{-1}\left(m_a^j R_2^{(j)}/R_1^{(j)}\right)$, $i = a, b, c$, $j = 1, 2, 3$. Is easy to see that each brane preserves $\mathcal{N}=2$ with the O6-plane if and only if $\vartheta_a^2 = \vartheta_a^3 = \vartheta_a$, etc... Notice that this is only possible if $\frac{m_c^2}{m_c^1} = \frac{m_a^2}{m_a^3}\frac{m_b^3}{m_b^1}$. This constraint is not present in models where one type of branes does not appear (for instance, see the models with a square quiver in section 6 and the phenomenological models from [24]).

If we index the possible vectors $\tilde{r}$ describing a SUSY as

$$\begin{aligned}
\tilde{r}_1 &= \pm\frac{1}{2}(-++-)\\
\tilde{r}_2 &= \pm\frac{1}{2}(+-+-)\\
\tilde{r}_3 &= \pm\frac{1}{2}(++--)\\
\tilde{r}_4 &= \pm\frac{1}{2}(----)
\end{aligned} \tag{4.12}$$

then we can represent the different SUSY's shared by the branes with the orientifold plane in table 2.

It is now easy to guess which supersymmetries will be preserved at each intersection. In general, an intersection ij will preserve the common supersymmetries that branes i and j preserve separately with the orientifold. This will imply that sectors with non-vanishing intersection number (chiral sectors) preserve one of the four SUSY's in (4.12), being six such intersections per supersymmetry. Sectors with vanishing intersection number (generally massive) contain a $\mathcal{N}=2$ subsector, coming from brane-brane* spectrum, and a $\mathcal{N}=0$ subsector, coming from branes of the same group. This general "$\mathcal{N}=2$" Q-SUSY structure can be expressed in a quiver-like manner as illustrated in figure 5. Notice that

Brane	Twist vector	SUSY preserved
a_1, (a_1^*)	$\pm(0, \vartheta_a, \vartheta_a)$	$\tilde{r}_2$, $\tilde{r}_3$
a_2, (a_2^*)	$\pm(0, \vartheta_a, -\vartheta_a)$	$\tilde{r}_1$, $\tilde{r}_4$
b_1, (b_1^*)	$\pm(\vartheta_b, 0, \vartheta_b)$	$\tilde{r}_1$, $\tilde{r}_3$
b_2, (b_2^*)	$\pm(-\vartheta_b, 0, \vartheta_b)$	$\tilde{r}_2$, $\tilde{r}_4$
c_1, (c_1^*)	$\pm(\vartheta_c, \vartheta_c, 0)$	$\tilde{r}_2$, $\tilde{r}_1$
c_2, (c_2^*)	$\pm(\vartheta_c, -\vartheta_c, 0)$	$\tilde{r}_3$, $\tilde{r}_4$

Table 2: Supersymmetry preserved by each brane with the O6-plane.

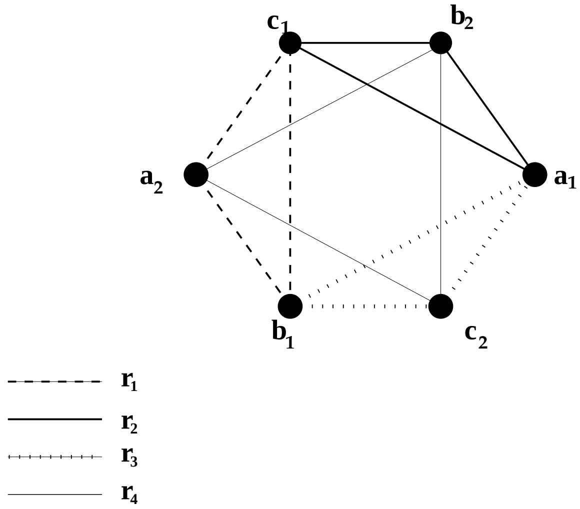

Figure 5: Quiver diagram corresponding to the general class of Q-SUSY models discussed in the text. The dots represent the six different types of branes (plus mirrors) in the models whereas the links represent the chiral intersections of those branes. There are four types of links corresponding to the different SUSY's the corresponding intersection preserves.

nodes in this hexagonal diagram represent gauge sectors with $\mathcal{N} = 2$,[11] and links represent chiral sectors where $\mathcal{N} = 1$ matter multiplets live. When there is no link between two nodes this signals a $\mathcal{N} = 0$ sector, generically massive. All this Q-SUSY structure concerns the open string sector of the theory, while it is embedded in the $\mathcal{N} = 4$ supersymmetry preserved by the closed string sector living in the bulk.

Different examples of Q-SUSY brane configurations may be obtained by deleting some of the nodes, as we will show in specific examples in section 6. A comment on superpotentials and Yukawa couplings in the general D-brane schemes from the quiver in figure 5 is in order. In explicit D6-brane constructions of Q-SUSY models some world-sheet instanton effects can communicate sectors with different $\mathcal{N} = 1$ SUSY's and yield explicit SUSY breaking even at the tree level. Indeed, for each triangle in figure 5 with sides of *the same type* (hence with chiral matter respecting the same SUSY) one may have in gen-

[11] Note that although the combined system of a brane and its mirror will respect only $\mathcal{N} = 2$, as long as they do not overlap the massless sector will fill $\mathcal{N} = 4$ representations.

eral trilinear superpotential couplings respecting the corresponding $\mathcal{N} = 1$. They come from a disk worldsheet with three intersecting branes at the boundary respecting the same $\mathcal{N} = 1$. However for each triangle in figure 5 of *different type* there will in general SUSY violating Yukawa couplings, since they will couple chiral multiplets respecting different supersymmetries. They will come from disk worldsheets with three intersecting branes at the boundary respecting *different* $\mathcal{N} = 1$. Note however that the actual presence and/or relevance of those SUSY-breaking Yukawa couplings will be model dependent. In some SUSY-quivers like the square quiver in section 6 such Yukawa couplings or superpotentials are absent (there are no subtriangles in the quiver). In other configurations, like the SUSY standard model of section 6 only SUSY superpotentials appear. More generally, in any model the size of some SUSY-breaking Yukawa couplings may be exponentially suppressed if the compact volume is large [22]. Thus, for example, in a realistic SM construction it is enough if the top and bottom quark Yukawa couplings are supersymmetric, since the other Yukawa couplings are very small and would not affect the stability of the Higgs mass in a sizable manner.

5. Effective field theory: gauge coupling constants and Fayet-Iliopoulos terms

We would like now to address some aspects of the effective low-energy field theory at the intersecting branes. Since we will study the local physics at the intersections our results will apply both to theories with full $\mathcal{N} = 1$ SUSY as well as to Q-SUSY theories. We will discuss in turn the gauge coupling constants and the Fayet-Iliopoulos terms. We will also discuss the structure of the NS tadpoles in Q-SUSY brane configurations.

5.1 Coupling constants and gauge kinetic functions

The gauge coupling constants may be computed from the DBI action but also, if a $\mathcal{N} = 1$ SUSY is preserved at the intersection, from the real part of a gauge kinetic function. The form of the latter may be obtained from holomorphicity once we know the imaginary part, which may be obtained from the known RR-couplings to gauge fields. We will show here how those two independent computations agree.

When looking at the low energy theory living on the world-volume of a single D6-brane we detect, from a four-dimensional point of view, a U(1) SYM theory whose gauge coupling constant is controlled by the tension of the brane on the compact dimensions. That is, by the length of the 3-cycle the brane wraps on the compact manifold $\mathcal{M}$.

$$\frac{1}{g_i^2} = \frac{M_s^3}{(2\pi)^4 \lambda} \|l_i\| \,, \tag{5.1}$$

where $M_s = \alpha'^{-\frac{1}{2}}$ is the string scale, λ is the string coupling, and $\|l_i\|$ is the 3-volume of the 3-cycle the brane is wrapping.[12]

[12] Notice that formula (5.1) is the correct expression for the $\mathrm{SU}(N_a)$ subgroup of $\mathrm{U}(N_a)$, with its generators in the fundamental representation normalized to unity. When computing FI-terms we will be dealing with a $\mathrm{U}(1)_a$ subgroup, whose generator will be taken to be Id_{N_a}. Both coupling constants are then related by $g^2_{(\mathrm{SU}(N_a))} = N_a g^2_{(\mathrm{U}(1)_a)}$.

In a supersymmetric field theory, though, the information concerning the gauge coupling constant should be encoded in the gauge kinetic function f_{ab} of the theory. This function enters on the supersymmetric lagrangian as

$$\mathcal{L}_g = -\frac{1}{4}\,\mathrm{Re}\, f_{ab} F_{a\ \mu\nu} F_b^{\mu\nu} + \frac{i}{4}\,\mathrm{Im}\, f_{ab} F_{a\ \mu\nu} \tilde{F}_b^{\mu\nu} \,, \tag{5.2}$$

from what we deduce

$$\mathrm{Re}\, f_{aa} = \frac{1}{g_a^2} = \frac{M_s^3}{(2\pi)^4 \lambda} \|l_a\| \,, \tag{5.3}$$

An important property of the gauge kinetic function is its holomorphicity. This means that, in a supersymmetric configuration, we should be able to express f as an holomorphic function on complex fields. What is more, we know what the real part of this function should look like, and it is also possible to compute the imaginary part by looking at the world-volume couplings of the form

$$\int_{M_4} \Phi_i F_a \wedge F_a \,, \tag{5.4}$$

that should arise in the dimensionally reduced effective theory. Here, Φ_i is a four-dimensional dimensionless scalar field. As an illustration of all this let us consider the orientifold case [9, 11, 12]. We will consider the T-dual theory to D6-branes at angles, which is D9-branes (type I theory) with magnetic fluxes. In this theory we have two ten-dimensional RR fields, C_2 and C_6, with world-volume couplings to the branes given by

$$\begin{gathered} i\mu_9 \frac{1}{2!} \int_{D9_a} C_6 \wedge \mathcal{F}_a^2 \,, \\ i\mu_9 \frac{1}{4!} \int_{D9_a} C_2 \wedge \mathcal{F}_a^4 \,, \end{gathered}$$

where $\mathcal{F}_a = 2\pi\alpha' F_a + B$ is the gauge invariant two-form flux living on brane a, and $\mu_9 = (2\pi)^{-9} M_s^{10}$ is the charge of the D9-brane under RR fields. When going to four dimensions, (5.4) couplings will arise by dimensional reduction. In [12] these couplings were computed for a factorizable brane a, and they turn out to be

$$\begin{gathered} n_a^1 n_a^2 n_a^3 \frac{i}{4\pi} \int_{M_4} C^0 \wedge F_a \wedge F_a \,, \\ n_a^I m_a^J m_a^K \frac{i}{4\pi} \int_{M_4} C^I \wedge F_a \wedge F_a \,, \end{gathered} \tag{5.5}$$

where the n's and m's stand for (fractional) wrapping numbers on each torus. C^0, C^I are four-dimensional scalar RR fields defined as

$$C^0 = \left(4\pi^2\alpha'\right)^{-3} \int_{T^6} C_6 \,, \qquad C^I = \left(4\pi^2\alpha'\right)^{-1} \int_{T_I^2} C_2 \,. \tag{5.6}$$

From these couplings we see that the imaginary part of the gauge kinetic function should be of the form

$$\mathrm{Im}(f_a) = \frac{1}{4\pi} \left(n_a^1 n_a^2 n_a^3 C^0 + \sum_I n_a^I m_a^J m_a^K C^I \right) , \tag{5.7}$$

whereas the real part should depend only on the volume of the brane

$$\mathrm{Re}(f_a) = \frac{M_s^3}{2\pi\lambda}\sqrt{\prod_{i=1}^{3}\left(\left(n_a^i R_1^{(i)}\right)^2 + \left(m_a^i R_2^{(i)}\right)^2\right)}\,. \tag{5.8}$$

Expressions (5.7) and (5.8) should be the real and imaginary part of the same holomorphic function, though they seem very different. (5.7) has a linear dependence on four RR fields, whereas (5.8) depends non-linearly on the NSNS moduli describing the complex structure of the torus. The solution to this apparent puzzle comes from the fact that the D6-brane a not always forms a supersymmetric system by itself, but has to preserve a supersymmetry with the O6-plane, lying on the homology cycle $[(1/\beta^1, 0)(1/\beta^2, 0)(1/\beta^3, 0)]$. If this brane and the O6-plane (or equivalently, if this brane and its mirror image a^*) preserve a supersymmetry, then the length of this brane can be expressed by a sum of fields, just as in (5.7). If, for instance, the twist vector with respect to the orientifold is given by $v_\vartheta = (\vartheta^1, \vartheta^2, \vartheta^1 + \vartheta^2, 0)$, $\vartheta^i > 0$, then by some basic trigonometry it can be shown that the volume of this brane a can be expressed as:

$$\begin{aligned}\frac{\|l_a\|}{(2\pi)^3} = {} & n_a^1 n_a^2 n_a^3 R_1^{(1)} R_1^{(2)} R_1^{(3)} + n_a^1 m_a^2 m_a^3 R_1^{(1)} R_2^{(2)} R_2^{(3)} + \\ & + m_a^1 n_a^2 m_a^3 R_2^{(1)} R_1^{(2)} R_2^{(3)} - m_a^1 m_a^2 n_a^3 R_2^{(1)} R_2^{(2)} R_1^{(3)}\end{aligned} \tag{5.9}$$

In order to properly compare both real and imaginary parts of the gauge kinetic function, let us first translate expression (5.9) into its T-dual counterpart. For this we must apply the T-duality transformations

$$R_2^{(I)} \leftrightarrow \frac{\alpha'}{R_2^{(I)}} \tag{5.10}$$

$$R_1^{(I)} \leftrightarrow R_1^{(I)} \tag{5.11}$$

$$\lambda \leftrightarrow \frac{\alpha'^{\,3/2}\lambda}{\prod_I R_2^{(I)}}\,. \tag{5.12}$$

After such transformations we can express the real part of the gauge kinetic function entirely in terms of a sum of NSNS fields

$$\mathrm{Re}(f_a) = n_a^1 n_a^2 n_a^3 S_f + n_a^1 m_a^2 m_a^3 U_f^1 + m_a^1 n_a^2 m_a^3 U_f^2 - m_a^1 m_a^2 n_a^3 U_f^3, \tag{5.13}$$

where we have defined

$$S_f \equiv \frac{M_s^6}{2\pi\lambda}\prod_{I=1}^{3} R_1^{(I)} R_2^{(I)}\,, \tag{5.14}$$

$$U_f^I \equiv \frac{M_s^2}{2\pi\lambda} R_1^{(I)} R_2^{(I)}\,. \tag{5.15}$$

Note that these correspond to the standard 4-D dilaton of toroidal compactifications and the three Kahler moduli of the three tori [36]. The subscript f stands for the fluxes (D9-brane) picture, where these NSNS four-dimensional moduli are relevant. In the T-dual picture of branes at angles, these same four-dimensional fields take the form

$$S_a \equiv \frac{M_s^3}{2\pi\lambda}\prod_{I=1}^{3} R_1^{(I)}\,, \tag{5.16}$$

$$U_a^I \equiv \frac{M_s^3}{2\pi\lambda} R_1^{(I)} R_2^{(J)} R_2^{(K)}\,. \tag{5.17}$$

Notice that from a four-dimensional viewpoint, we cannot distinguish from which of the T-dual systems we are compactifying. Fields (5.14), (5.15) and (5.16), (5.17) are exactly the same when talking about low energy physics in four dimensions. We can, then, delete the subscript a or f in order to describe our field theory. We should only take into account which field are we working with when doing a computation where the geometry of the compactification is relevant. Any of the T-dual choices should give us the same result.

It is now easy to express the gauge kinetic function as an holomorphic function on the relevant fields, namely as a sum of four complex fields, whose real part consist of some complex structure field and the imaginary part of some RR untwisted field. In general, we find that given a preserved $\mathcal{N}=1$ SUSY described by the vector $\tilde{r} = \frac{1}{2}(\epsilon^1, \epsilon^2, \epsilon^3, \epsilon^4)$ belonging to (4.12), then the gauge kinetic function can be expressed as:

$$f_a = n_a^1 n_a^2 n_a^3 \tilde{S} + \sum_I n_a^I m_a^J m_a^K \tilde{U}^I\,, \tag{5.18}$$

where the complex fields $\tilde{S}, \tilde{U}$ are given by

$$\tilde{S} = S + \frac{i}{4\pi} C^0\,, \tag{5.19}$$

$$\tilde{U}^I = -U^I \epsilon^J \epsilon^K + \frac{i}{4\pi} C^I\,. \tag{5.20}$$

Thus, we have defined four complex scalar fields that will appear in our effective field theory description of our compactification. It is interesting to notice that the definition of these fields depends on which specific supersymmetry is preserved by our brane a and its mirror. Indeed, departing from the $\mathcal{N}=4$ bulk supersymmetry preserved by our plain type I theory, we have defined four $\mathcal{N}=1$ independent subalgebras represented by four independent spinors (4.12). Each of these $\mathcal{N}=1$ SUSY's can be expressed in terms of one of these vectors $\tilde{r}$, in turn encoded in terms of the ϵ's appearing in (5.20). To sum up, given a brane preserving a $\mathcal{N}=1$ SUSY with its mirror, we can express the gauge kinetic function and the Kahler potential (which define the supersymmetric low energy effective action) in terms of some complex scalar fields. The form of these fields is inherited by bulk superfields, as expected, but with some relative signs ϵ^i depending on the specific $\mathcal{N}=1$ preserved by the brane. We will call each choice of signs in (5.20) a different SUSY prescription, that tell us how RR and NSNS bulk field relate in order to enter a chiral $\mathcal{N}=1$ multiplet as a complex scalar field. Notice that each of the branes appearing in our hexagonal models from section 4 do not only preserve $\mathcal{N}=1$, but $\mathcal{N}=2$. Then, two different $\tilde{r}$ vectors could be considered. Our complex superfields are so described by two different SUSY prescriptions which are, in fact, the same, since only one of the fields $\tilde{U}^I$ couples to the brane.

5.2 Fayet-Iliopoulos terms

We saw in the previous section how for particular choices of the complex structure moduli one can obtain an unbroken $\mathcal{N}=1$ SUSY at an intersection. Now, if we modify slightly the value of those moduli supersymmetry will be broken. It is reasonable to expect that the effect of this breaking will be approximately described by the turning on of a Fayet-Iliopoulos term in the theory. We will describe now in some detail how this happens and how the scalar fields at the intersection get masses from the FI-terms. Those masses agree as expected with the ones obtained from the string mass formulae eq. (4.5).

In the previous subsection we also showed the gauge kinetic function that should describe the effective field theory of a D6-brane preserving a supersymmetry with the O6-plane. If we define a twist vector v_ϑ between the orientifold and this brane, the condition for preserving such supersymmetry can be reformulated as that one of the 'scalars' in (4.5) should become massless,[13] thus defining a SUSY wall in the 3-dimensional parameter space $(\vartheta^1, \vartheta^2, \vartheta^3)$. Since we have four different scalars, four SUSY walls appear, forming a tetrahedron. The structure of this tetrahedron and its physical consequences have been studied in [12, 25]. Briefly stated, when standing inside the tetrahedron all of the scalars (4.5) are massive, signaling that no supersymmetry is preserved by both branes. When standing outside this tetrahedron one of such scalars will become tachyonic, and this indicates that the system is unstable as it stands.

When a system composed of two factorizable branes a and b lies in one of these tetrahedron walls they are in a marginal stability (MS) wall. The concept of marginal stability is in fact more general, and can be applied to branes wrapping SLAGs in general cycles [31]. As discussed in [37], when crossing such a wall a tachyon may appear at the intersection of the two branes, signaling an unstability against the recombination of these two branes into a single one. This final brane will wrap a SLAG whose homology class is determined by the sum of the RR charges of the two previous branes. Supersymmetry will be restored under recombination, while one of the U(1)'s will become massive. At the other side of the MS wall, this recombination is not favoured locally in terms of difference of tensions, so the pair of branes will not recombine into one. We will then have a non-supersymmetric configuration with gauge group $\mathrm{U}(1)_a \times \mathrm{U}(1)_b$. As noted in [37], this behaviour can be understood from a Field Theory viewpoint by including

[13] Such scalars are in fact non-existent in the low energy theory, since no open string lives on the intersection of a brane and the orientifold plane. We could, in turn, consider the scalars arising in the $D6_a D6_{a^*}$ sector, whose twist vector is $2v_\vartheta$ and thus have two times the mass of these fictitious scalars. Strictly speaking, all the considerations made in this section regarding the system brane-orientifold should be translated to the system brane-mirror brane.

a Fayet-Iliopolous term $D\xi$ in the effective lagrangian, so that the potential energy becomes

$$V_{FI}(\phi) = \frac{1}{2g^2}\left(|\phi|^2 + \xi\right)^2 , \tag{5.21}$$

where ξ is the transversal separation from the MS wall, and ϕ are the lightest complex scalars living at the intersection. For a T-dual description of this phenomenon see [38].

When considering a full configuration of branes, where several FI terms arise, we would expect several contributions to the potential energy, each of them given by

$$V_{FI,a}(\phi_i) = \frac{1}{2g_a^2}\left(\sum_i q_a^i|\phi_i|^2 + \xi_a\right)^2 . \tag{5.22}$$

Here, ξ_a represents the FI-term associated to $\mathrm{U}(1)_a$, and ϕ_i are all the scalar fields charged under $\mathrm{U}(1)_a$ with charge q_a^i. It should be possible to compute these FI terms from an effective field theory description of the gauge theory living on these branes. Let us consider, as before, an orientifold compactification and a brane a preserving a SUSY $\tilde{r}$ with the O6-plane. When separating a bit from the SUSY wall, the supersymmetric field theory should break. This supersymmetry breaking should be understood as some FI-terms in this theory. As noted in [39], the FI terms in a SUSY theory can be deduced from the couplings of the form

$$\frac{Q_a^i}{2\pi\alpha'}\int_{M_4} B_2^i \wedge F_a , \tag{5.23}$$

that arise in the compactified four-dimensional theory. Here B_2^i are four-dimensional antisymmetric fields (dual to the RR C-scalars discussed above) and the dimensionless coeficcient Q_a^i represents how the brane a couples to such field. These couplings are the mediators of the Green-Schwarz anomaly cancellation mechanism .[14] Just as done above, we can compute which couplings of type (5.23) appear from a four-dimensional viewpoint when dimensionally reducing the couplings (5.5). As found in [12], they turn out to be

$$\begin{aligned}
&\frac{1}{4\pi^2\alpha'}N_a m_a^1 m_a^2 m_a^3 \int_{M_4} B_2^0 \wedge F_a , \\
&\frac{1}{4\pi^2\alpha'}N_a m_a^I n_a^J n_a^K \int_{M_4} B_2^I \wedge F_a ,
\end{aligned}$$

where the N_a factors arise from the normalization of the $\mathrm{U}(1)_a$ subgroup of $\mathrm{U}(N_a)$ (see [40, 10]).

These two-forms B_2 living in our four-dimensional field theory are again defined by partially integrating the couplings (5.5) on T^6

$$B_2^0 = C_2 , \qquad B_2^I = (4\pi^2\alpha')^{-2}\int_{(T_J^2)\times(T_K^2)} C_6 . \tag{5.24}$$

[14] Note that massless anomaly-free U(1)'s will have no FI-terms. However anomaly-free U(1)'s which become massive through a $B \wedge F$ coupling may in general get a FI away from the SUSY wall.

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Both scalars (5.6) and two-forms (5.24) are four-dimensional RR fields related to each other by hodge duality in four dimensions:

$$dC^0 = - * dB_2^0 \tag{5.25}$$
$$dC^I = - * dB_2^I \,. \tag{5.26}$$

By supersymmetric field theory arguments, we would expect Fayet-Iliopolous terms of the form

$$D_a \frac{\xi_a}{g_a^2} = D_a \left(\frac{\partial \mathcal{K}}{\partial V_a}\right)_{V=0} , \tag{5.27}$$

where V_a is the vector multiplet associated to the massive $\mathrm{U}(1)_a$, and $\mathcal{K}$ the Kahler potential, whose gauge invariant expression in these toroidal compactifications is given by

$$\mathcal{K} = \frac{M_{\text{Planck}}^2}{8\pi}\left(-\log\left(\tilde{S} + \tilde{S}^* - \sum_a Q_a^0 V_a\right) - \sum_{i=1}^{3}\log\left(\tilde{U}^i + \tilde{U}^{i*} - \sum_a Q_a^i V_a\right)\right). \tag{5.28}$$

Substituting in (5.27) under the SUSY prescription (5.19), (5.20) we obtain

$$\begin{aligned}\frac{\xi_a}{g_a^2} &= \frac{M_P^2}{32\pi^2}\frac{(2\pi)^3\lambda N_a}{\mathrm{Vol}(T^6)M_s^3}\prod_{i=1}^{3}\epsilon^i\left((2\pi)^3\prod_{i=1}^{3}\epsilon^i m_a^i R_2^{(i)}(2\pi)^3\sum_{i=1}^{3}\epsilon^i m_a^i n_a^j n_a^k R_2^{(i)}R_1^{(j)}R_1^{(k)}\right)\\ &= \frac{M_s^5 N_a}{(2\pi)^5\lambda}\prod_{i=1}^{3}\epsilon^i\left((2\pi)^3\prod_{i=1}^{3}\epsilon^i m_a^i R_2^{(i)} - (2\pi)^3\sum_{i=1}^{3}\epsilon^i m_a^i n_a^j n_a^k R_2^{(i)}R_1^{(j)}R_1^{(k)}\right),\end{aligned} \tag{5.29}$$

where we have used $M_P^2 = 8M_s^8\mathrm{Vol}(T^6)/(2\pi)^6\lambda^2$ [36]. For convenience we have written this expression in terms of "D6-branes at angles" geometric moduli, One can check that the term in brackets vanishes at the SUSY wall, thus giving the expected behaviour for a FI-term. It turns out that it has a simple dependence on the separation parameter δ from the supersymmetric case. To illustrate this, let us take our previous example and variate the complex structures in order to have a twist vector $v_\vartheta = (\vartheta^1, \vartheta^2, \vartheta^1 + \vartheta^2 + \delta, 0)$, $\vartheta^i > 0$. Then, for small δ, our approximate SUSY is still given by the vector $\tilde{r} = \frac{1}{2}(+,+,-,-)$ and after some trigonometry we find that our expression becomes

$$\frac{\xi_a}{g_a^2} = -\frac{M_s^5}{(2\pi)^5\lambda}N_a\|l_a\|\sin(\pi\delta) = -\frac{sin(\pi\delta)}{(2\pi)g_a^2}M_s^2 \tag{5.30}$$

where we have used (5.1), and taken into account that $g_a^2 \equiv g^2_{\mathrm{U}(1)_a}$ (see footnote 11). In the limit of small separation from the SUSY wall, that is when $\delta \ll 1$, is where we expect our field theory approximation to be valid. In this limit we can approximate our FI-term by

$$\alpha'\xi_a \approx -\frac{\delta}{2}\,. \tag{5.31}$$

In order to compute the mass of a scalar living at the intersection of two branes, let us take two D6-branes $D6_a$ and $D6_b$ whose separation from the same SUSY wall is given by δ_a and δ_b, respectively. By looking at the effective potential (5.22), we would expect the mass of this scalar to be given by

$$\alpha' m_{ab}^2 = \alpha'(-q_a\xi_a - q_b\xi_b) \approx \frac{1}{2}(\delta_a - \delta_b)\,. \tag{5.32}$$

Notice that this result is in agreement with the masses for the scalars lying at a intersection obtained from the string mass formulae in (4.5). Thus we see that, for small deviations from a SUSY configuration, the masses of the scalars at an intersection may be understood as coming from a Fayet-Iliopoulos term.

5.3 Application to $\mathcal{N} = 1$ SUSY models

Let us now apply our Field Theory results to a $\mathcal{N} = 1$ SUSY model, as those presented in [14]. Recall that in those $\mathbb{Z}_2 \times \mathbb{Z}_2$ models branes were related to the O6-plane by a rotation from the same subgroup SU(3) $\subset$ SU(4). Namely, they had the twist vectors (4.10) which, in our hexagonal construction from figure 5, translates into a restriction of such general models to those containing the SUSY triangle formed by a_2, b_2 and c_2 branes, who share the SUSY $\tilde{r}_4 = \frac{1}{2}(+,+,+,+)$. Using formulae (5.19), (5.20), we see that the volume of any of such branes can be expressed as

$$\|l_a\| = \prod_{i=1}^{3} n_a^i R_1^{(i)} - \sum_{i=1}^{3} n_a^i m_a^j m_a^k R_1^{(i)} R_2^{(j)} R_2^{(k)} . \tag{5.33}$$

Let us consider then a full configuration of D6-branes preserving this $\mathcal{N} = 1$ SUSY with the orientifold plane. We can again look at the potential derived from the DBI action by dimensional reduction

$$V = \frac{T_6}{\lambda} \left(\sum_a N_a \|l_a\| - \|l_{\mathrm{ori}}\| \right) , \tag{5.34}$$

where the contribution from mirror branes need not be taken into in this particular computation. Since we are dealing with a supersymmetric configuration this quantity should vanish in a consistent compactification. In these particular $\mathbb{Z}_2 \times \mathbb{Z}_2$ models, the negative tension of the orientifold planes is given by

$$\|l_{\mathrm{ori}}\| = 16 \prod_{i=1}^{3} R_1^{(i)} + 16 \sum_{i=1}^{3} R_1^{(i)} \beta^j R_2^{(j)} \beta^k R_2^{(k)} . \tag{5.35}$$

The conditions for having no NS tadpoles simply translate into a vanishing potential, so they are

$$\sum_a N_a \; n_a^1 n_a^2 n_a^3 \; = \; 16 \tag{5.36}$$

$$\sum_a N_a \; n_a^i m_a^j m_a^k \; = \; -16 \beta^j \beta^k \tag{5.37}$$

which are just the RR tadpoles cancellation conditions derived in [14]. The results from previous sections also apply to this kind of models, having a gauge kinetic function for each brane whose holomorphicity or SUSY prescription (5.19), (5.20) is the same for every brane.

Brane	Twist vector	approx. SUSY
a_2, (a_2^*)	$\pm(0, \vartheta_a + \delta^2, -\vartheta_a)$	$\tilde{r}_1$, $\tilde{r}_4$
b_2, (b_2^*)	$\pm(-\vartheta_b - \delta^1, 0, \vartheta_b)$	$\tilde{r}_2$, $\tilde{r}_4$
c_2, (c_2^*)	$\pm(\vartheta_c + \alpha^1\delta^1, -\vartheta_c - \alpha^2\delta^2, 0)$	$\tilde{r}_3$, $\tilde{r}_4$

Table 3: Small SUSY breaking for a $\mathcal{N} = 1$ triangle.

As a second application, let us rederive the FI terms appearing in (5.22) from a different viewpoint. The contribution to this effective potential coming from ξ_a^2/g_a^2 should come from the closed strings modes that participate in (5.34). Indeed, we should be able to compute this contribution by considering the value of (5.34) slightly away from the SUSY wall. Now, notice that a $\mathcal{N} = 1$ SUSY model where the complex structures have been varied still satisfies

$$\|l_{\mathrm{ori}}\| = \sum_a N_a \|l_a\| \cos \pi\delta_a \tag{5.38}$$

where δ_a is the separation of each brane from the SUSY wall. The potential (5.34) is then given by

$$V = \frac{T_6}{\lambda} \sum_a N_a \|l_a\| \left(1 - \cos \pi\delta_a\right) = \frac{M_s^4}{(2\pi)^2} \sum_a \frac{1}{g^2_{\mathrm{U}(1)_a}} \left(1 - \cos \pi\delta_a\right), \tag{5.39}$$

where we have used the explicit form of T_6 and formula (5.1). For small separation we again can approximate this expression, obtaining

$$V \approx \frac{1}{2} \sum_a \frac{1}{g^2_{\mathrm{U}(1)_a}} \left(\frac{\delta_a}{2\alpha'}\right)^2, \tag{5.40}$$

which again gives the expected behaviour for a potential from a FI term.

Finally, let us study how the system behaves when the $\mathcal{N} = 1$ supersymmetry of these kind of models is slightly broken. We will focus on a fairly general subsector of a SUSY (or Q-SUSY) compactification where we have a SUSY triangle with branes of type a_2, b_2 and c_2 only. In order to slightly break $\mathcal{N} = 1$ SUSY, we can make a small variation on the complex structure. Let us, for instance, place ourselves in a point of the moduli space of complex structures where the SUSY condition is satisfied. We can make a small variation on the radii of the first and second tori, which induces a small variation on the angles the branes make. Such variations are parametrized by small δ's defined as $\vartheta_a^2 = \vartheta_a + \delta^2$, and $\vartheta_b^1 = \vartheta_b + \delta^1$. Notice that this already fixes the c sector departure from SUSY breaking. In general, we can parametrize the small Q-SUSY breaking by these two terms δ^i, $i = 1, 2$. Table 2 is now modified to table 3.

In table 3, α^1 and α^2 are two proportionality factors arising from the different length of the branes. Their expression, valid for small δ, is given by $\alpha^1 = \frac{\mathrm{sen} 2\vartheta_c}{\mathrm{sen} 2\vartheta_b}$ and $\alpha^2 = \frac{\mathrm{sen} 2\vartheta_c}{\mathrm{sen} 2\vartheta_a}$. Let us fix a hierarchy on the angle's values. Take, for instance, $\vartheta_a < \vartheta_b < \vartheta_c$. We can easily compute the mass spectrum arising from the lightest scalars living at the intersections. This is done in table 4, where we show the corresponding twist vector, and we compute

Intersection	Twist vector	approx. SUSY	mass^2
a_2b_2	$(-\vartheta_b-\delta^1,-\vartheta_a-\delta^2,\vartheta_b+\vartheta_a)$	$\tilde{r}_4$	$\frac{1}{2}(\delta^1+\delta^2)$
$a_2b_2^*$	$(\vartheta_b+\delta^1,-\vartheta_a-\delta^2,-\vartheta_b+\vartheta_a)$	$\tilde{r}_4$	$\frac{1}{2}(-\delta^1+\delta^2)$
a_2c_2	$(\vartheta_c+\alpha^1\delta^1,-\vartheta_c-\vartheta_a-(\alpha^2+1)\delta^2,\vartheta_a)$	$\tilde{r}_4$	$\frac{1}{2}(\alpha^1\delta^1-(\alpha^2+1)\delta^2)$
$a_2c_2^*$	$(-\vartheta_c-\alpha^1\delta^1,\vartheta_c-\vartheta_a+(\alpha^2-1)\delta^2,\vartheta_a)$	$\tilde{r}_4$	$\frac{1}{2}(-\alpha^1\delta^1+(\alpha^2-1)\delta^2)$
b_2c_2	$(\vartheta_c+\vartheta_b+(\alpha^1+1)\delta^1,-\vartheta_c-\alpha^2\delta^2,-\vartheta_b)$	$\tilde{r}_4$	$\frac{1}{2}(-(\alpha^1+1)\delta^1+\alpha^2\delta^2)$
$b_2c_2^*$	$(-\vartheta_c+\vartheta_b-(\alpha^1-1)\delta^1,\vartheta_c+\alpha^2\delta^2,-\vartheta_b)$	$\tilde{r}_4$	$\frac{1}{2}((\alpha^1-1)\delta^1-\alpha^2\delta^2)$

Table 4: Small SUSY breaking at intersections. $\mathcal{N}=1$ triangle.

the square mass of such scalar particles, the superpartners of each massless fermion living at these intersections. Recall that, as shown above, these masses may be understood as coming from induced FI-terms for the three U(1) fields in these models.

Notice that, for any value of the SUSY breaking parameters δ^1, δ^2 a tachyon always appears at some intersection. This can be easily seen if we notice that, for this hierarchy of angles ($\vartheta_a^j<\vartheta_b^j<\vartheta_c^j$), identities between sparticles masses such as the following hold:

$$m^2(a_2b_2)+m^2(a_2b_2^*)+m^2(a_2c_2)+m^2(a_2c_2^*)=0\,, \tag{5.41}$$

which clearly implies that one of the scalars living at these four intersections must become tachyonic when breaking the $\mathcal{N}=1$ SUSY by varying the complex structure. It is important to stress that a general $\mathcal{N}=1$ model does not have necessarily to include such a triangle. However, the appearance of a tachyon at some intersection when slightly varying the complex structure from the supersymmetric case seems a general feature of any $\mathcal{N}=1$ compactification satisfying tadpole cancellation.

Note that these tachyons are nothing but an indication that the initial configuration of branes is unstable and that the two intersecting branes will fuse into a single one minimizing the energy [14]. In the process the gauge symmetry will be broken and the rank reduced. Thus, as usually happens in all known $D=4$ string constructions the presence of a FI-term does not signal SUSY-breaking but just the existence of a nearby vacuum which is again supersymmetric. We will now see that in specific Q-SUSY models that is not in general the case, i.e., the presence of FI-terms does not necessarily give rise to tachyonic masses for any scalars and hence SUSY is actually broken at the intersections.

5.4 Q-SUSY models: NS-tadpoles and FI-terms

Unlike the $\mathcal{N}=1$ case, the scalar potential for the S,U^I Neveu-Schwarz fields does not vanish in Q-SUSY models, i.e., there are uncancelled NS tadpoles. This is expected, the theory being non-supersymmetric. However, one can see that the form of this scalar potential is particularly simplified in the case of Q-SUSY models, compared to a general

N_i	(n_i^1, m_i^1)	(n_i^2, m_i^2)	(n_i^3, m_i^3)	v_i
N_{a_1}	$(1/\beta^1, 0)$	$(n_{a_1}^2, m_{a_1}^2)$	$(n_{a_1}^3, m_{a_1}^3)$	$(0, \vartheta_{a_1}^2, \vartheta_{a_1}^3)$
N_{a_2}	$(1/\beta^1, 0)$	$(n_{a_2}^2, m_{a_2}^2)$	$(n_{a_2}^3, -m_{a_2}^3)$	$(0, \vartheta_{a_2}^2, -\vartheta_{a_2}^3)$
N_{b_1}	$(n_{b_1}^1, m_{b_1}^1)$	$(1/\beta^2, 0)$	$(n_{b_1}^3, m_{b_1}^3)$	$(\vartheta_{b_1}^1, 0, \vartheta_{b_1}^3)$
N_{b_2}	$(n_{b_2}^1, -m_{b_2}^1)$	$(1/\beta^2, 0)$	$(n_{b_2}^3, m_{b_2}^3)$	$(\vartheta_{b_2}^1, 0, \vartheta_{b_2}^3)$

Table 5: Example of D6-brane wrapping numbers giving rise to a square quiver Q-SUSY model. We are again taking $n_i^j, m_i^j \geq 0$, $i = a, b$, $j = 1, 2, 3$.

non-SUSY toroidal model. In particular, we find that the potential is linear in the S, U NS fields and for some simple cases the four-dimensional dilaton (S) tadpole even vanishes once the RR-tadpoles vanish. Let us first check this point in a simple example.

Let us consider a subclass of Q-SUSY models where no branes of type c appear. We will call such models quadrilateral or square quiver models (see figure 6 and next section). To be concrete, we will consider a configuration where four different branes appear each of a different kind, as shown in table 5.

We will suppose that this is a RR tadpole-free configuration, which amounts to the following restrictions

$$\begin{aligned} \sum_{i=1}^{2} \left(\frac{1}{\beta^1} N_{a_i} n_{a_i}^2 n_{a_i}^3 + \frac{1}{\beta^2} N_{b_i} n_{b_i}^1 n_{b_i}^3 \right) &= 16 \\ N_{a_1} m_{a_1}^2 m_{a_1}^3 - N_{a_2} m_{a_2}^2 m_{a_2}^3 &= 0 \\ N_{b_1} m_{b_1}^1 m_{b_1}^3 - N_{b_2} m_{b_1}^2 m_{b_2}^3 &= 0 \end{aligned} \tag{5.42}$$

The length of the branes composing such a model is easily computed from (5.19), (5.20)

$$\begin{aligned} \|l_a\| &= (2\pi)^3 \frac{1}{\beta^1} \left(n_a^2 n_a^3 R_1^{(1)} R_1^{(2)} R_1^{(3)} + m_a^2 m_a^3 R_1^{(1)} R_2^{(2)} R_2^{(3)} \right) \\ \|l_b\| &= (2\pi)^3 \frac{1}{\beta^2} \left(n_b^1 n_b^3 R_1^{(1)} R_1^{(2)} R_1^{(3)} + m_b^1 m_b^3 R_2^{(1)} R_1^{(2)} R_2^{(3)} \right), \end{aligned}$$

where we index $a = a_1, a_2$, same for b. When substituting these quantities in the potential (5.34), we get (again not including mirror branes)

$$\begin{aligned} V &= \frac{T_6}{\lambda} \left(\sum_a N_a \|l_a\| - \|l_{\text{ori}}\| \right) \\ &= \frac{M_s^7}{(2\pi)^3 \lambda} \left(\frac{1}{\beta^1} N_a m_a^2 m_a^3 R_1^{(1)} R_2^{(2)} R_2^{(3)} + \frac{1}{\beta^2} N_b m_b^1 m_b^3 R_2^{(1)} R_1^{(2)} R_2^{(3)} \right) \\ &= \frac{M_s^4}{(2\pi)^2} \left(\frac{1}{\beta^1} N_a m_a^2 m_a^3 U^1 + \frac{1}{\beta^2} N_b m_b^1 m_b^3 U^2 \right). \end{aligned} \tag{5.43}$$

Notice that, as promised, the dependence of U^1, U^2 is linear on these fields. This is a general characteristic of Q-SUSY models and not of this particular example. Note also that only two out of four NSNS fields appear in brackets, S and U^3 being absent. There

Brane	Twist vector	approx. SUSY
a_1, (a_1^*)	$\pm(0, \vartheta_{a_1} + \delta^2, \vartheta_{a_1})$	$\tilde{r}_2$, $\tilde{r}_3$
a_2, (a_2^*)	$\pm(0, \vartheta_{a_2} + \alpha^2\delta^2, -\vartheta_{a_2})$	$\tilde{r}_1$, $\tilde{r}_4$
b_1, (b_1^*)	$\pm(\vartheta_{b_1} + \delta^1, 0, \vartheta_{b_1})$	$\tilde{r}_1$, $\tilde{r}_3$
b_2, (b_2^*)	$\pm(-\vartheta_{b_2} - \alpha^1\delta^1, 0, \vartheta_{b_2})$	$\tilde{r}_2$, $\tilde{r}_4$

Table 6: Small Q-SUSY breaking for a square quiver model. In this general example we are allowing $\vartheta_{b_1} \neq \vartheta_{b_2}$. There are some proportionality factors defined as $\alpha^1 \equiv \frac{\sin 2\vartheta_{b_2}}{\sin 2\vartheta_{b_1}}$, $\alpha^2 \equiv \frac{\sin 2\vartheta_{a_2}}{\sin 2\vartheta_{a_1}}$.

is no NS tadpole for the $D = 4$ dilaton field S, and there is only one NS tadpole left corresponding to a linear combination of the U^1 and U^2 fields. Thus, the structure of NS tadpoles in this class of models is substantially simplified compared to generic non-SUSY orientifold models.

Let us now do the same exercise we did for the case of $\mathcal{N} = 1$ models concerning FI-terms but for this Q-SUSY configuration. We will show how in this Q-SUSY example turning on a FI-term does not necessarily lead to gauge symmetry breaking (but unbroken SUSY) as happenned in the $\mathcal{N} = 1$ case. Rather one can get unbroken gauge symmetry (no tachyons) but broken SUSY at the intersections.

In order to study how the system behaves when the Quasi-supersymmetry is slightly broken, we can make a small variation of the complex structures just as we did with the SUSY triangle. Thus, we will first consider a complex structure such that $\vartheta^2_{a_i} = \vartheta^3_{a_i}$ and $\vartheta^1_{b_i} = \vartheta^3_{b_i}$, $i = 1, 2$ and then perform a small change on the quotient of radii such that there is small departure from these equalities, again parametrized by δ's. The corresponding twist vectors are shown in table 6.

Just as in the SUSY triangle, we can again compute the masses of the lightest scalars at the eight relevant intersections. For this will consider again a hierarchy of angles, which we choose to be $\vartheta_{a_1} < \vartheta_{a_2} < \vartheta_{b_1} < \vartheta_{b_2}$. The square mass for the corresponding qsuperpartners of the massless fermions is computed in table 7.

Is easy to see that if we choose variations δ^1, δ^2 that satisfy

$$\delta^1 > \delta^2 > \alpha^1\delta^1 > 0 \qquad \text{and} \qquad \alpha^1\delta^1 > \alpha^2\delta^2 \,, \tag{5.44}$$

then all scalars appearing at an intersection have positive mass^2. Although we have performed this explicit computation for a specific hierarchy of angles, we expect this qualitative behaviour to hold in general cases. Thus this is a remarkable difference compared to the $\mathcal{N} = 1$ systems: in Q-SUSY configurations the FI-terms do not necessarily trigger gauge symmetry breaking.[15] Thus FI-terms do actually break $\mathcal{N} = 1$ SUSY locally (in addition to the overall SUSY-breaking appearing at the loop level).

[15]Note that SUSY-breaking by FI-terms in a Q-SUSY model will thus not obey the Ferrara-Girrardello-Palumbo type of sum-rules [41] which precluded the construction of phenomenological models with tree-level SUSY-breaking in the early days of SUSY-phenomenology.

Intersection	Twist vector	approx. SUSY	mass^2
a_1b_1	$(\vartheta_{b_1}+\delta^1,-\vartheta_{a_1}-\delta^2,\vartheta_{b_1}-\vartheta_{a_1})$	$\tilde{r}_3$	$\frac{1}{2}(\delta^1-\delta^2)$
$a_1b_1^*$	$(-\vartheta_{b_1}-\delta^1,-\vartheta_{a_1}-\delta^2,-\vartheta_{b_1}-\vartheta_{a_1})$	$\tilde{r}_3$	$\frac{1}{2}(\delta^1+\delta^2)$
a_1b_2	$(-\vartheta_{b_2}-\alpha^1\delta^1,-\vartheta_{a_1}-\delta^2,\vartheta_{b_2}-\vartheta_{a_1})$	$\tilde{r}_2$	$\frac{1}{2}(-\alpha^1\delta^1+\delta^2)$
$a_1b_2^*$	$(\vartheta_{b_2}+\alpha^1\delta^1,-\vartheta_{a_1}-\delta^2,-\vartheta_{b_2}-\vartheta_{a_1})$	$\tilde{r}_2$	$\frac{1}{2}(\alpha^1\delta^1+\delta^2)$
a_2b_1	$(\vartheta_{b_1}+\delta^1,-\vartheta_{a_2}-\alpha^2\delta^2,\vartheta_{b_1}+\vartheta_{a_2})$	$\tilde{r}_1$	$\frac{1}{2}(\delta^1+\alpha^2\delta^2)$
$a_2b_1^*$	$(-\vartheta_{b_1}-\delta^1,-\vartheta_{a_2}-\alpha^2\delta^2,-\vartheta_{b_1}+\vartheta_{a_2})$	$\tilde{r}_1$	$\frac{1}{2}(-\delta^1+\alpha^2\delta^2)$
a_2b_2	$(-\vartheta_{b_2}-\alpha^1\delta^1,-\vartheta_{a_2}-\alpha^2\delta^2,\vartheta_{b_2}+\vartheta_{a_2})$	$\tilde{r}_4$	$\frac{1}{2}(\delta^1+\delta^2)$
$a_2b_2^*$	$(\vartheta_{b_2}+\alpha^1\delta^1,-\vartheta_{a_2}-\alpha^2\delta^2,-\vartheta_{b_2}+\vartheta_{a_2})$	$\tilde{r}_4$	$\frac{1}{2}(-\alpha^1\delta^1+\alpha^2\delta^2)$

Table 7: Small Q-SUSY breaking for a square quiver model. The specific hierarchy of angles $\vartheta_{a_1}<\vartheta_{a_2}<\vartheta_{b_1}<\vartheta_{b_2}$ has been chosen in order to compute the last column of this table.

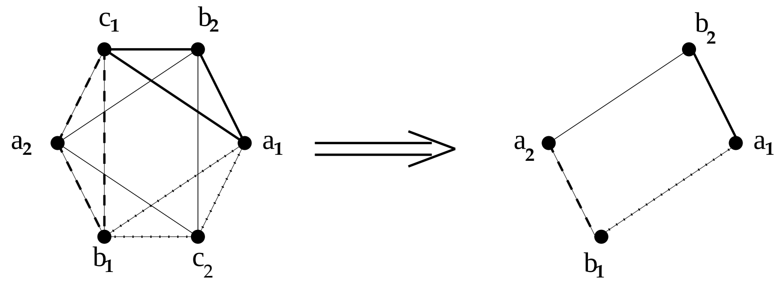

Figure 6: Square quiver Q-SUSY model with four group factors. The dots represent the four different stacks of branes (plus mirrors) in the model whereas the links represent the chiral intersections of those branes. There are four types of links corresponding to the different SUSY's.

6. Some explicit models

Starting with the general hexagonal structure discussed in section 4 one can construct a variety of interesting Q-SUSY models. In particular, one can obtain simple models by deleting some of the nodes in the hexagon in figure 5. We discuss here a couple of examples.

i. Q-SUSY models from a square quiver

   In specific models not all six types of D6-branes $(a_1,b_1,c_1,a_2,b_2,c_2)$ need to be present. For example, one can have a square type of quiver with only branes of type a_1,a_2,b_1,b_2 (see figure 6). The presence of both types of branes a_1,b_1 and their relatives a_2,b_2 which contribute with opposite sign to some of the RR tadpoles, allow for the construction of theses models without further addition of other branes to cancel tadpoles.

   In fact such type of structure is the one appearing in the realistic models of [12] in which the starting gauge group is $U(3)\times U(2)\times U(1)\times U(1)$. These four group factors correspond to the four vertices in the square quiver in figure 6. After three U(1)'s get massive through a generalized Green-Schwarz mechanism, only the SM group

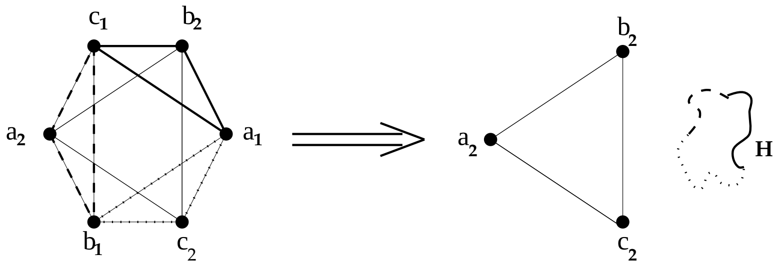

Figure 7: Model with an anomaly-free subsector respecting $\mathcal{N} = 1$ supersymmetry. RR tadpole cancellation conditions require in this case the presence of extra sources of RR flux. In particular a non-factorizable brane H can be added which has no intersection with the "visible" $\mathcal{N} = 1$ SUSY subsector.

survives and three generations of quarks and leptons are obtained at the intersections. Although these models are in general not supersymmetric, it has been recently shown that a subset of these models has Q-SUSY for appropriate choices of the complex structure moduli. This class of Q-SUSY models yielding the SM is discussed in some detail in a separate paper [24] and we direct the reader to that reference for more details.

ii. A Q-SUSY Standard Model with $\mathcal{N} = 1$ supersymmetry in the visible sector

Another interesting class of theories may be obtained starting with three of the six types of D6-branes considered above. One can consider theories in which a certain subsector respects a given $\mathcal{N} = 1$ supersymmetry. For example, consider three sets of intersecting branes of type a_2, b_2, c_2 (see figure 7). The intersections of these three branes respect the same SUSY, $\tilde{r}_4$, thus this subsector of the theory is fully supersymmetric. In a simple toroidal (non-orbifold) setting as the one here this cannot be the full story, we already mentioned in section 3 that such a configuration would have RR-tadpoles. Cancellation of those tadpoles requires extra sources of RR-flux. In the orbifold models of [14] that was achieved by the presence of three extra orientifold planes in the system. In our case we will achieve that by adding a non-factorizable D6-brane chosen precisely to cancel the remaining RR-tadpoles.[16] Let us call it the H-brane. It is easy to convince oneself that, in all generality, if the subsystem of the intersecting branes has an anomaly-free spectrum, the extra non-factorizable brane H will have no net intersection with the original ("visible") a_2, b_2, c_2 brane system. Thus, in general fields transforming both under the "visible branes" and the H-brane quantum numbers will be massive, typically of order the string/compactification scale. In this way we will have a $\mathcal{N} = 1$ supersymmetric visible sector formed by branes of types a_2, b_2, c_2 and a sort of "hidden sector" with $\mathcal{N} = 0$ SUSY in general.

As an example consider four stacks of branes with wrapping numbers and multiplicities given in table 8. It is easy to check that for appropriate choices of the tori

[16] Alternatively one can add explicit RR fluxes as in e.g. [42] in order to cancel tadpoles. See in particular [43].

brane type	N_i	(n_i^1, m_i^1)	(n_i^2, m_i^2)	(n_i^3, m_i^3)
a_2	$N_a = 3$	$(1, 0)$	$(3, 1/2)$	$(3, -1/2)$
b_2	$N_b = 2$	$(1, -1)$	$(2, 0)$	$(1, 1/2)$
c_2	$N_c = 1$	$(0, 1)$	$(0, -1)$	$(2, 0)$
a_2'	$N_d = 1$	$(1, 0)$	$(3, 1/2)$	$(3, -1/2)$

Table 8: Wrapping numbers of a three generation SUSY-SM with $\mathcal{N} = 1$ SUSY locally.

Intersection	Matter fields		Q_a	Q_b	Q_c	Q_d	Y
$(a_2 b_2)$	q_L	$2(3, 2)$	1	-1	0	0	1/6
$(a_2 b_2^*)$	Q_L	$(3, 2)$	1	1	0	0	1/6
$(a_2 c_2)$	D_R	$3(\bar{3}, 1)$	-1	0	1	0	1/3
$(a_2 c_2^*)$	U_R	$3(\bar{3}, 1)$	-1	0	-1	0	-2/3
$(b_2 a_2$')	l	$2(1, 2)$	0	-1	0	1	-1/2
$(b_2 a_2^*$')	L	$(1, 2)$	0	1	0	1	-1/2
$(c_2 a_2$')	E_R	$3(1, 1)$	0	0	1	-1	1
$(c_2 a_2^*$')	N_R	$3(1, 1)$	0	0	-1	-1	0
$(b_2 c_2)$	H	$2(1, 2)$	0	1	-1	0	-1/2
$(b_2 c_2^*)$	$\bar{H}$	$2(1, 2)$	0	1	1	0	1/2

Table 9: Chiral spectrum of the SUSY SM in the text.

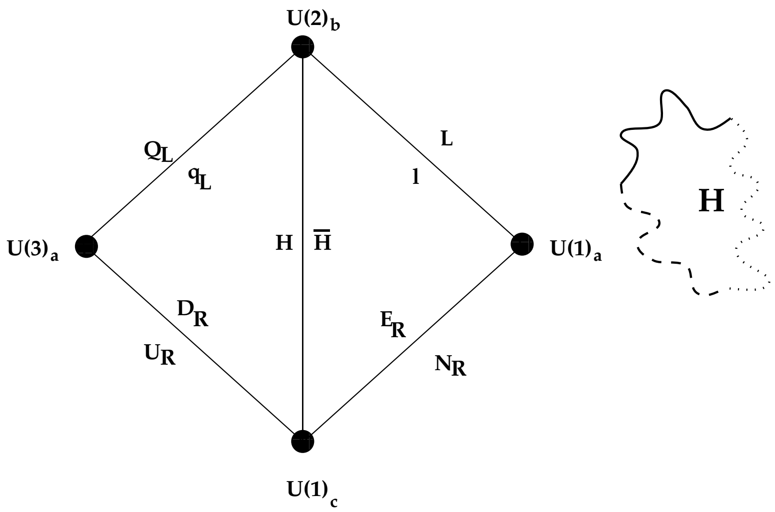

Figure 8: Quiver of the three-generation SM with $\mathcal{N} = 1$ SUSY at the local level discussed in the text.

complex structure there is an unbroken SUSY ($\tilde{r}_4$) at all brane intersections, as we discussed in section 4. These four sets of branes form a subsector of the theory with the same unbroken $\mathcal{N} = 1$ SUSY. Cancellation of RR-tadpoles requires the presence of an extra, in general non-factorizable D6-brane H with vanishing intersection with our SUSY subsector (see figure 8). The chiral fields at each intersection may be computed and one finds the chiral spectrum in table 9. This spectrum is just the one of the Minimal SUSY Standard Model with two Higgs sets. Although in principle there are four U(1) fields in the "visible sector" two of them are anomalous and become massive. The two anomaly-free U(1)'s are in fact $(B-L)$ and the usual hypercharge.

Note that in these theories the Higgs doublet scalar masses are protected against quadratic divergences up to two loops, as explained in section 2. Although the "visible sector" of the model has $\mathcal{N}=1$ SUSY, there is a non-SUSY massive sector from strings stretching between the visible sector and the non-factorizable brane system H. Loops as in figure 2(b) will give two-loop contributions to the Higgs masses of order $\alpha_2/(4\pi)M_s \sim 3\times 10^{-3}M_s$.[17] Thus the scale of electroweak scale symmetry breaking may be naturally small as long as $M_s \leq 30\,TeV$. On the other hand gaugino masses will get masses at the one-loop level from diagrams as in figure 3.[18] Electroweak symmetry breaking may proceed in a radiative way as in the MSSM from the one-loop coupling of Higgs fields to the heaviest quarks [45].

As will be explained elsewhere in more detail [24] the standard model Higgs mechanism in intersecting brane models has a nice geometrical interpretation in terms of recombination of intersecting branes. Thus e.g. a vev for the Higgs fields H and $\bar{H}$ would correspond to the recombination of three branes:

$$b_2 + b_2^{*\prime} + c_2 \to f \tag{6.1}$$

into a single one f. Here b_2 and b_2' denote the two branes giving rise to the $\mathrm{U}(2)_L$ gauge interactions. This recombination proceeds by a smoothing out of the intersections, which is controlled by the vevs of the Higgs fields. In the final configuration there are only three stacks of branes a_2, a_2' and f and the gauge group is $\mathrm{SU}(3)_c\times\mathrm{U}(1)_{em}$.[19] Once the Higgs fields get a vev, the quarks and leptons in the standard model get masses. In the language of brane recombination this can be verified by noting that the final recombined brane f has no intersection with the other two, no chiral fermions are left. Thus for example, $I_{a_2f} = I_{a_2b_2} + I'_{a_2b_2^*} + I_{a_2c} = 2+1-3 = 0$, there are no chiral quarks left.

[17] Interestingly enough, in case we add explicit RR fluxes to cancel tadpoles there would be in general no such massive fields [43] charged with respect to the SM interaction. Thus the model would rather look like standard $\mathcal{N}=1$ SUSY models with a SUSY breaking hidden sector. In this case the $\mathcal{N}=1$ SUSY of the visible sector would protect the scalars and we could then increase the string scale well above the electroweak scale.

[18] Note that the structure of the low-energy SUSY spectrum would be relatively similar to the models with gauge mediated SUSY-breaking [44]. Thus one does not expect important FCNC effects from the sparticle sector.

[19] Actually in this particular example there is an extra unbroken U(1) related to $B-L$ which was already present in the initial configuration. The Higgs fields are neutral under it and do not give it a mass. It may become massive if a right-handed sneutrino mass gets a vev [24]. A third U(1) gets massive from the presence of $B\wedge F$ couplings.

7. Final comments and conclusions

In this paper we have presented a class of $D = 4$ chiral field theories with interesting loop stability properties. They can be depicted as quivers in which we have extended SUSY gauge theories at the nodes and bifundamental chiral multiplets with respect to different $\mathcal{N} = 1$ SUSY's at the links. Altogether the theories are not supersymmetric but the scalars only feel that SUSY has been broken at two loops. This property may be interesting in order to understand the stability of a hierarchy between a fundamental scale $\sim$ 10-100 TeV and the weak scale of order 0.1 TeV.

It is important to realize that some level of low-energy supersymetry may be already needed to understand the precision LEP data [7]. This need is independent of the solution proposed for the classical gauge hierarchy problem between the weak scale and the Planck scale. Even alternative solutions like a low string scale slightly above the weak scale have eventually to face this modest fine-tuning problem. On the other hand full $\mathcal{N} = 1$ supersymmetry is more than what we actually need in models with low string scale. In this context Q-SUSY models may be of phenomenological relevance.

Field theories with these properties naturally appear in toroidal compactifications of type II string theory with intersecting D6-branes wrapping 3-cycles in the torus. For particular choices of the tori radii one gets chiral SUSY multiplets at the different intersections, but generically preserving different $\mathcal{N} = 1$ subgroups, yielding the searched Q-SUSY structure. We have studied certain aspects of the effective lagrangian at those intersections, which apply both to complete $\mathcal{N} = 1$ SUSY configurations and the Q-SUSY configurations here introduced. In particular we compute the gauge coupling constants from the Born-Infeld action and from the effective lagrangian point of view (using holomorphicity and anomalous RR-couplings) and show their agreement. We also study small perturbations of the complex structure moduli away from the SUSY point. We observe that, as expected, this produces SUSY-breaking by FI-terms. We compute those FI-terms and show how the masses given to scalars coincides with the computation from string mass formulae.

In section 6 we construct several explicit D-brane models. In particular one can construct theories which subsectors respecting $\mathcal{N} = 1$ locally but in which massive $\mathcal{N} = 0$ sectors induce SUSY-breaking in loops. A particular interesting model is constructed with a massless spectrum strikingly similar to the MSSM. This gives us an explicit example of a D-brane theory in which the Higgs mass is stable up to two loops, providing a stable hierarchy between a string scale of order 30 TeV and the weak scale. The usual Higgs mechanism has a nice geometrical interpretation in terms of brane recombination: the branes of the electroweak group recombine into a single brane which is related to electromagnetism. At the same time the quarks and leptons become massive in the process. We leave more phenomenological aspects of this and other explicit models for a separate paper [24].

One point we have not addressed explicitly in this paper is the stability of the considered specific D6-brane configurations. The models are non-supersymmetric and hence NS-NS tadpoles are expected, which show an instability of the theory *in a flat background*. These may be understood as coming from the existence of a non-vanishing tree-level scalar potential for the real part of the moduli S, U^I. The Q-SUSY models turn out to have a particularly simple structure for this potential compared to generic toroidal constructions. The potential is linear in those NS fields and in particular cases some of the NS tadpoles (but not all) can be shown to cancel if the corresponding RR tadpole does. On the other hand one may perhaps get rid of the instability a la Fischler-Susskind [46], by redefining the closed string background. This redefinition has been shown to lead to warped metrics in some particular simple examples [47]. In realistic models to study the viability of such a procedure may be quite complicated.

Other related issue is that of obtaining adequate $D = 4$ gravity, with a large Planck scale compared to the string scale in the toroidal D-brane constructions. One cannot get a large Planck scale by taking a large transverse volume, because in the models considered there is not a volume which is transverse to all the branes simultaneously. In this context perhaps a gravity localization a la Randall-Sundrum could be at work. The presence of warping factors from a non-vanishing NS scalar potential could be relevant if that possibility is present.

Notice finally that the class of type II orientifold configurations here studied have $\mathcal{N} = 4$ supergravity in the bulk ($\mathcal{N} = 8$ in the case of toroidal compactifications). The SUSY partners of the graviton will only get their mass from the SUSY-breaking effects on the branes. It would be interesting to study possible consequences of such approximate extended supergravity in the bulk (see e.g. [48] and references therein).

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